MIT 6.5620/6.875/18.425
Foundations of Cryptography

Lecture 21: Remote RAM Computation

November 22, 2023

Elephant in the room:
I’m not Vinod

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  - Examples: file storage, medical study with many patients, analytics on user data
- **Common solution**: Store your data and run computation on a **remote server.**

Basic Setup

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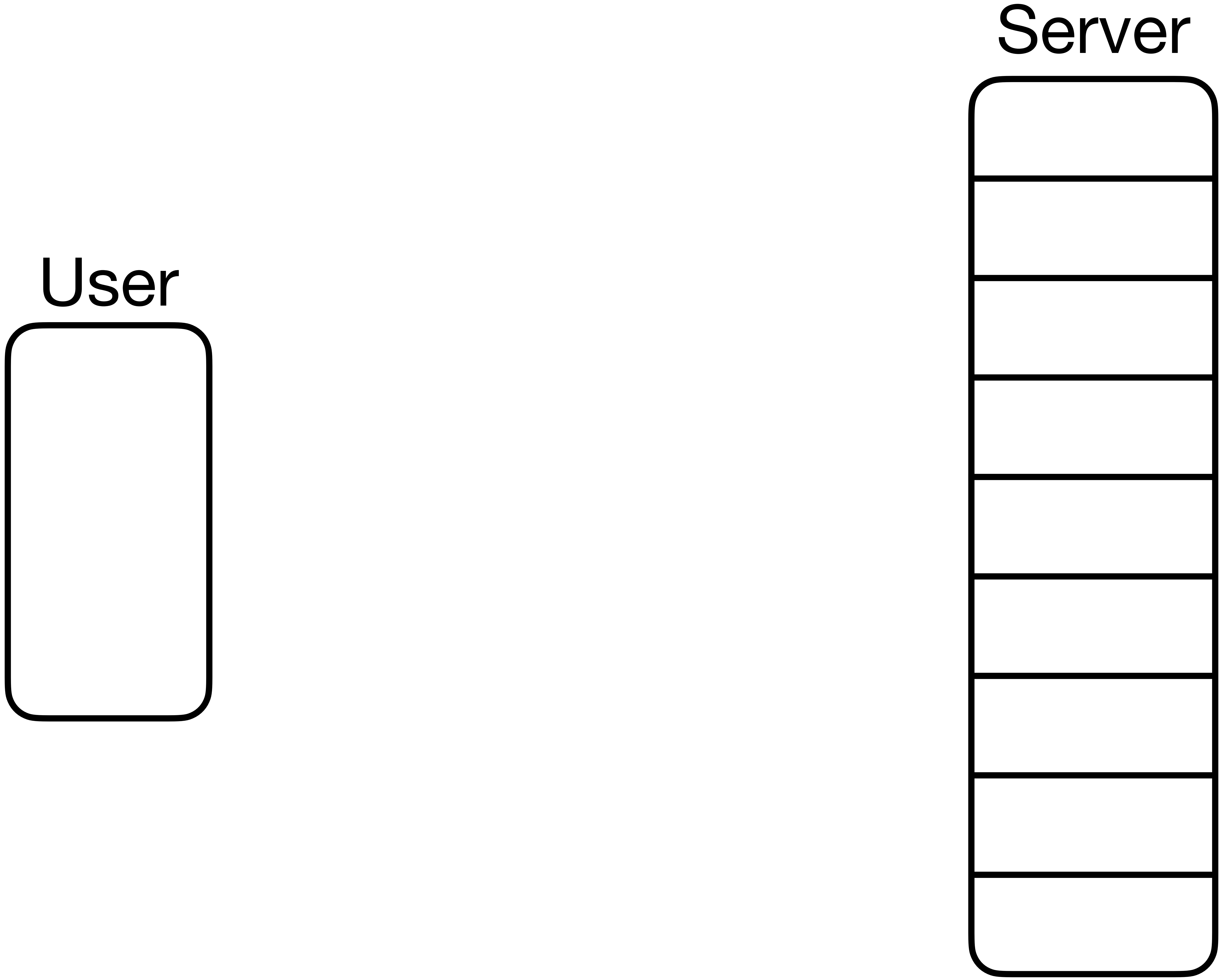

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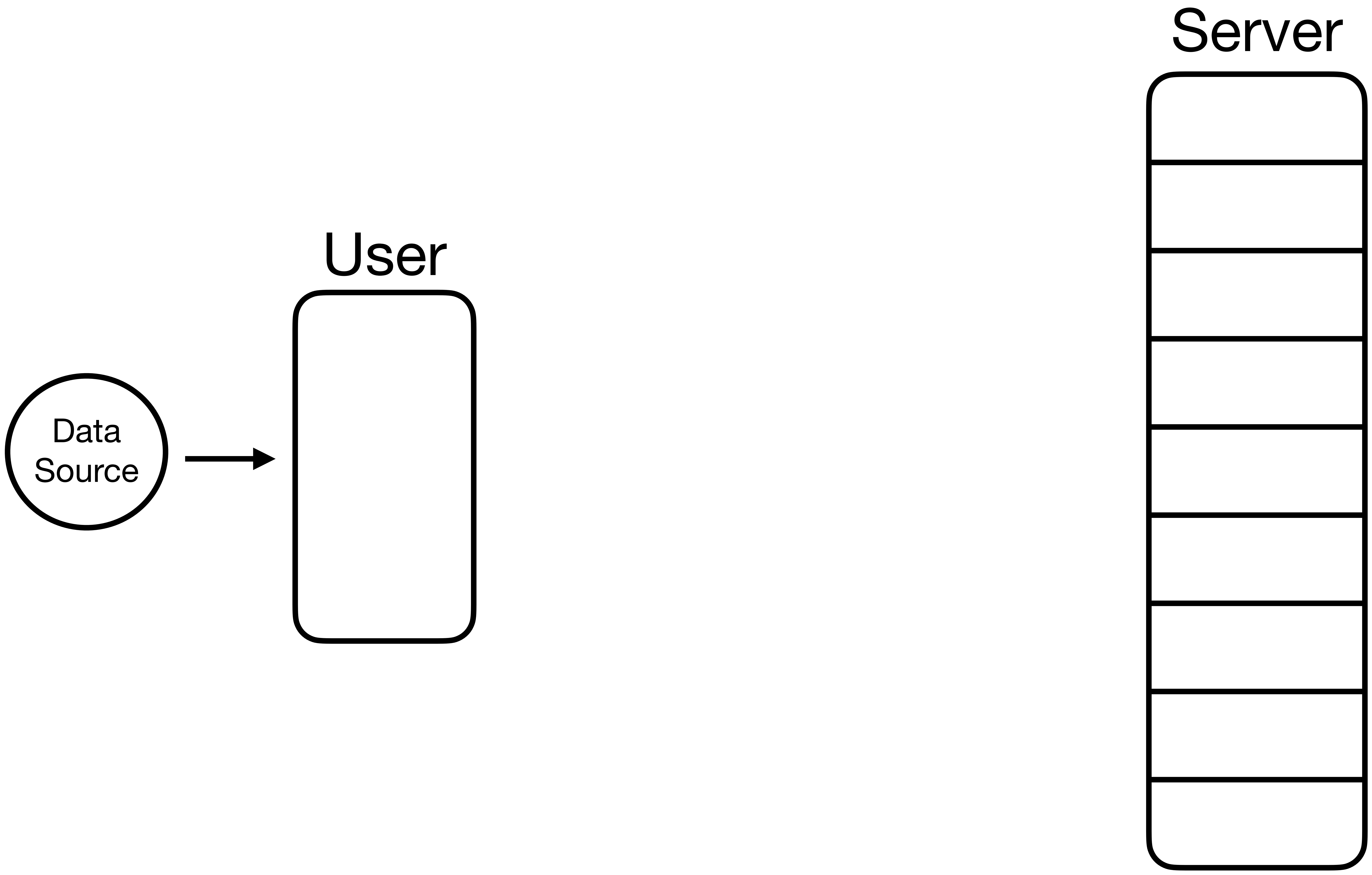

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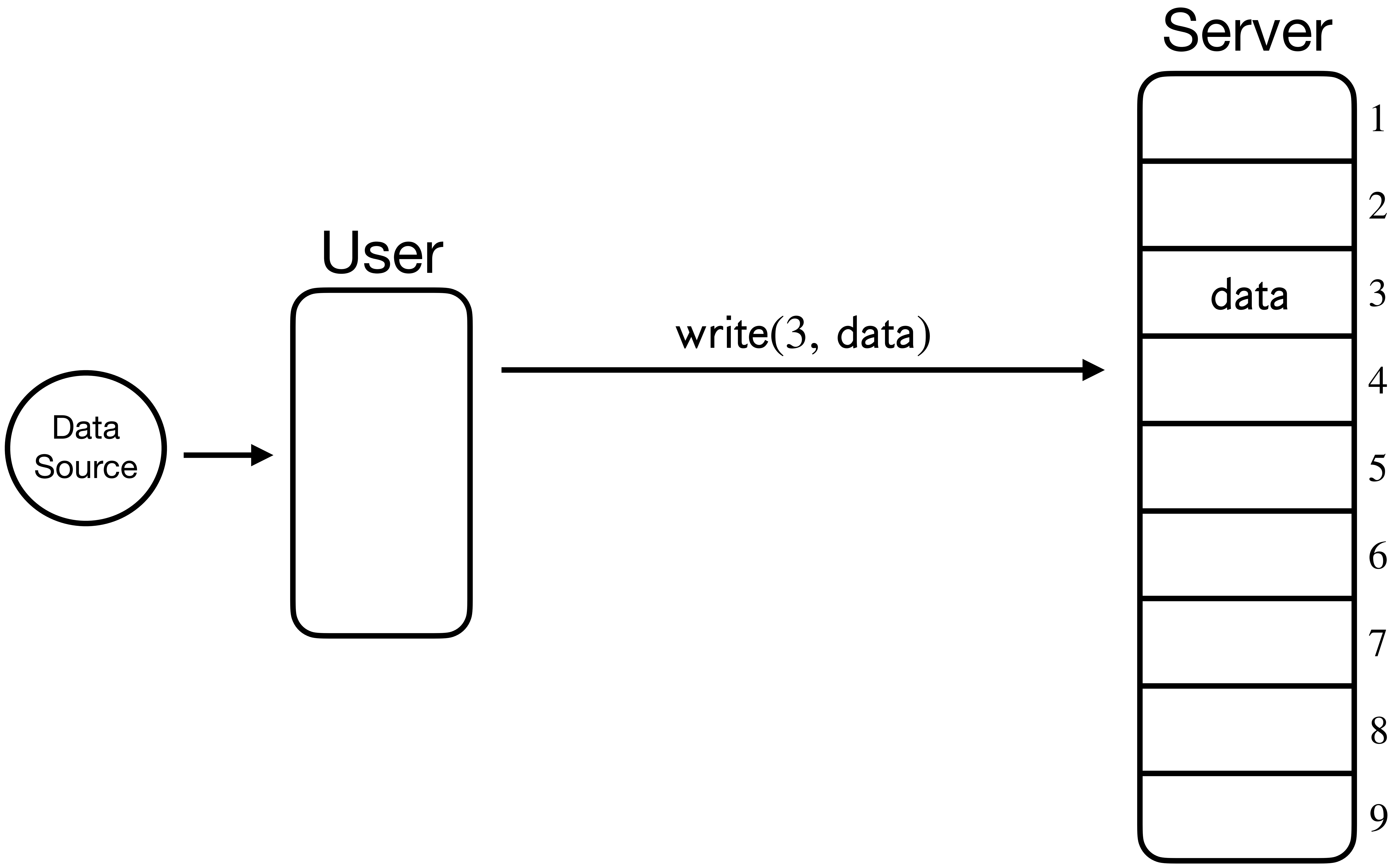

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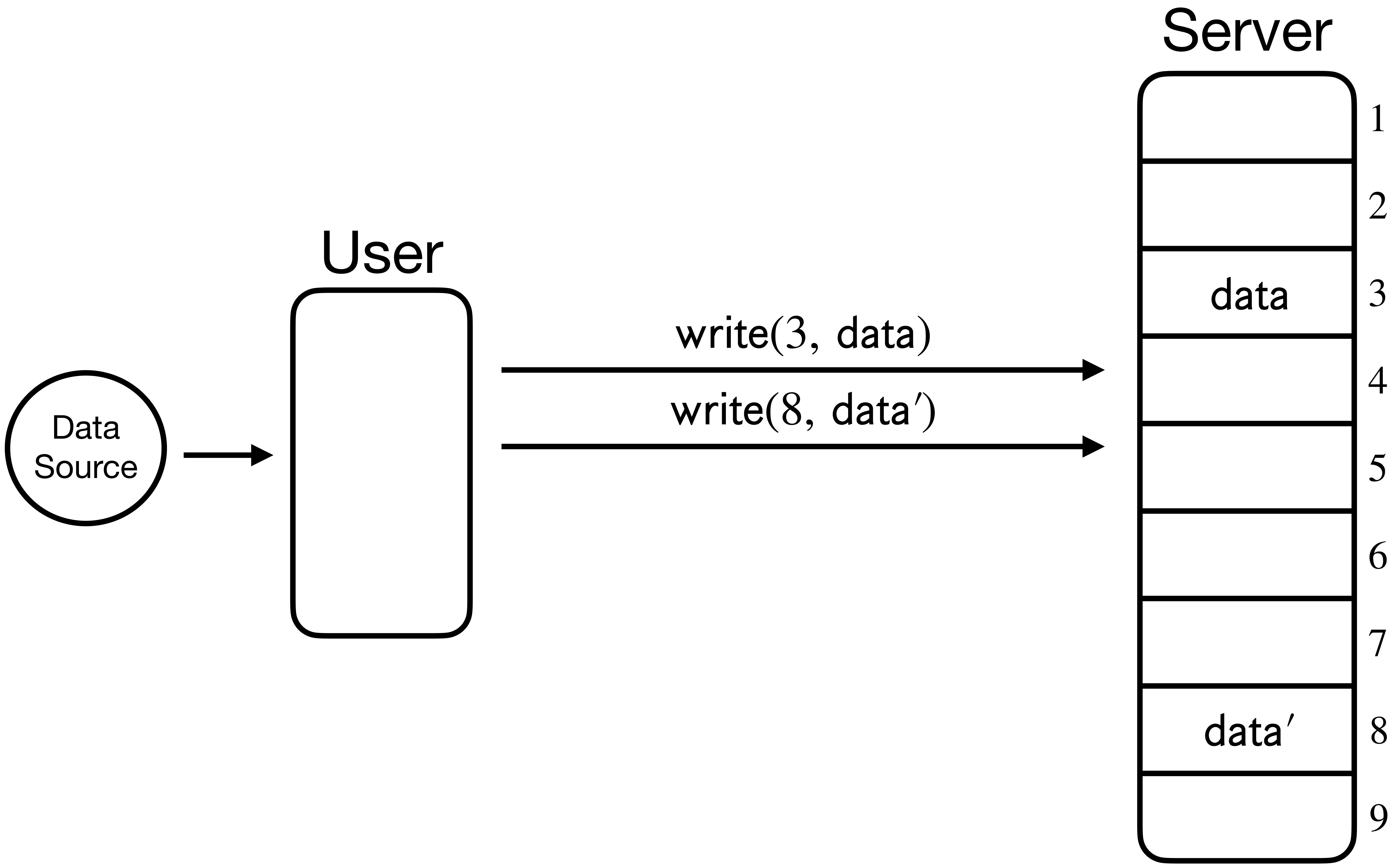
Server
User
Data Source
write(3, data)
write(8, data′)
data
data′
1
2
3
4
5
6
7
8
9

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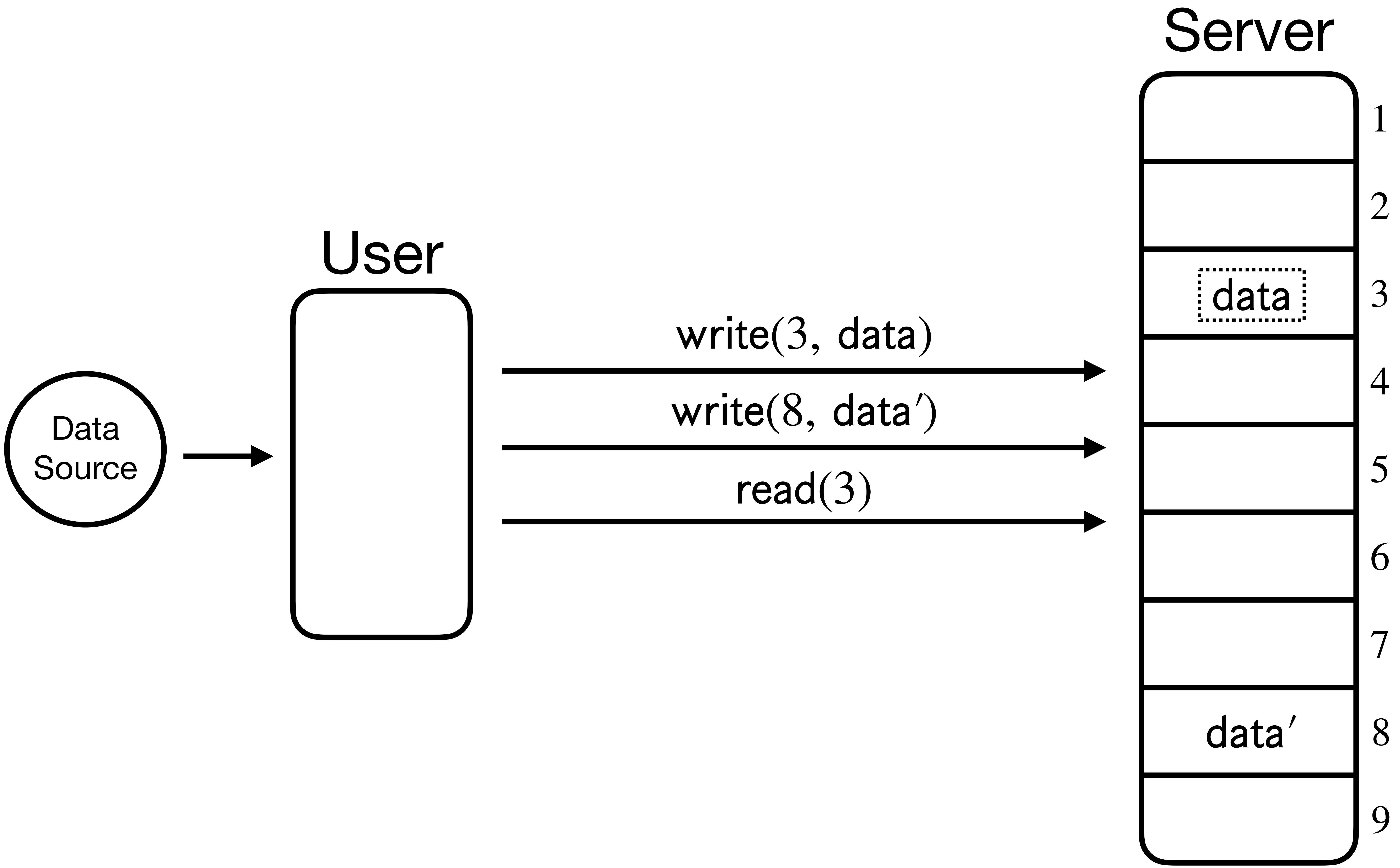

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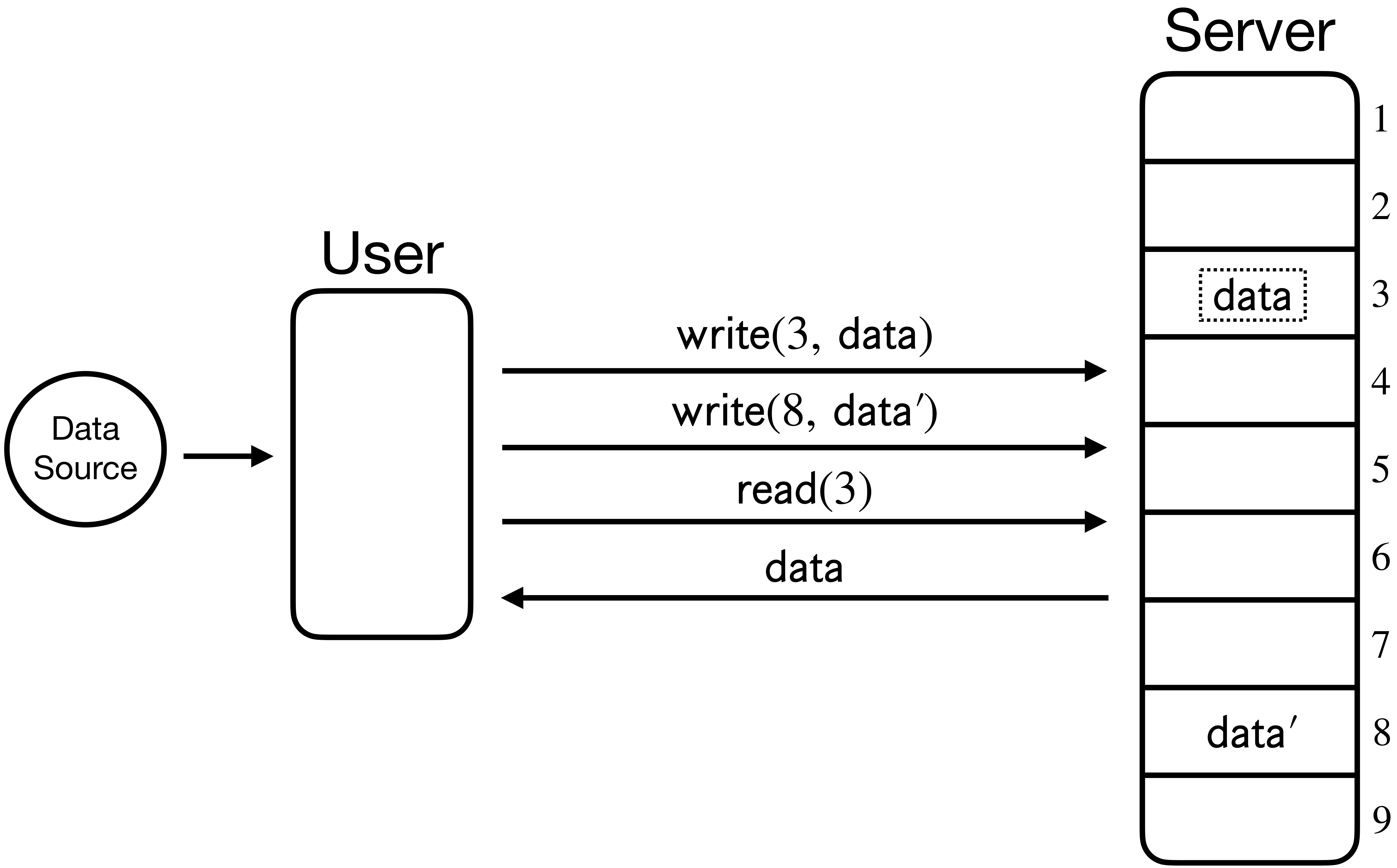
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  - …right? Do you trust them?
- Why shouldn't we trust the server?
- What are we trying to prevent?

Trust Issue 1: Integrity

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    - **Yes!** (At some cost – we'll see.)
    - (Adversary will learn length of computation / amount of data, but that's it.)

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  - **Still doable!** (At some cost – we'll see if time permits.)

Solutions to These Issues: Terminology

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3. Privacy and integrity issue: **Maliciously Secure ORAM**

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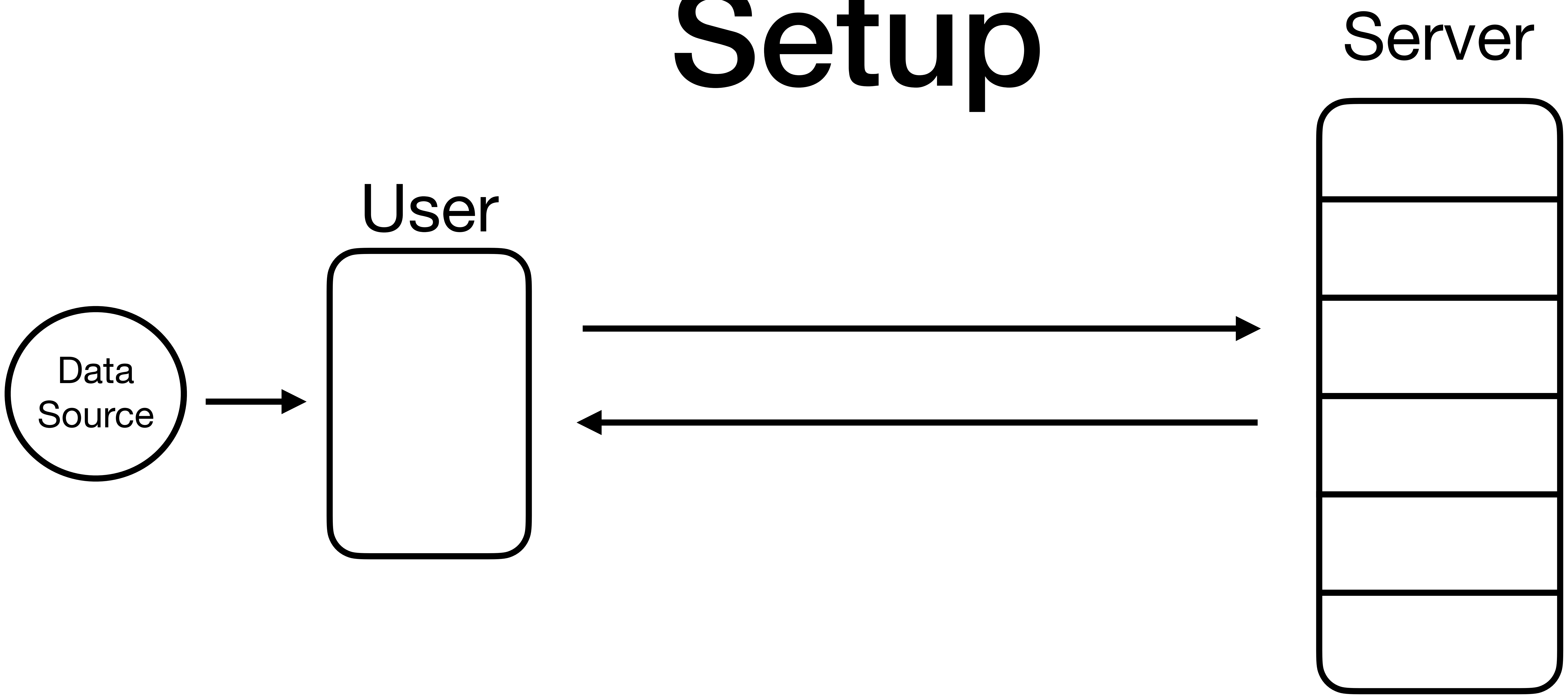

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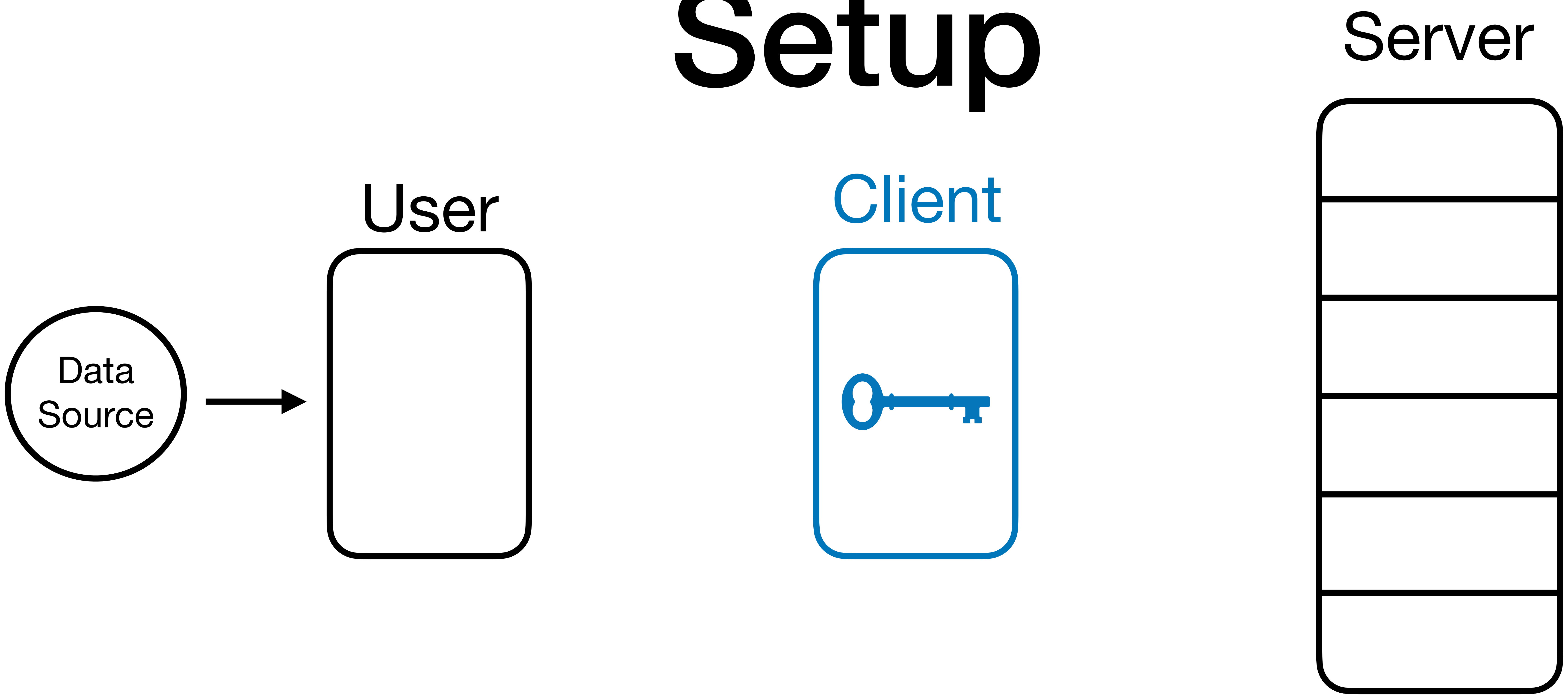

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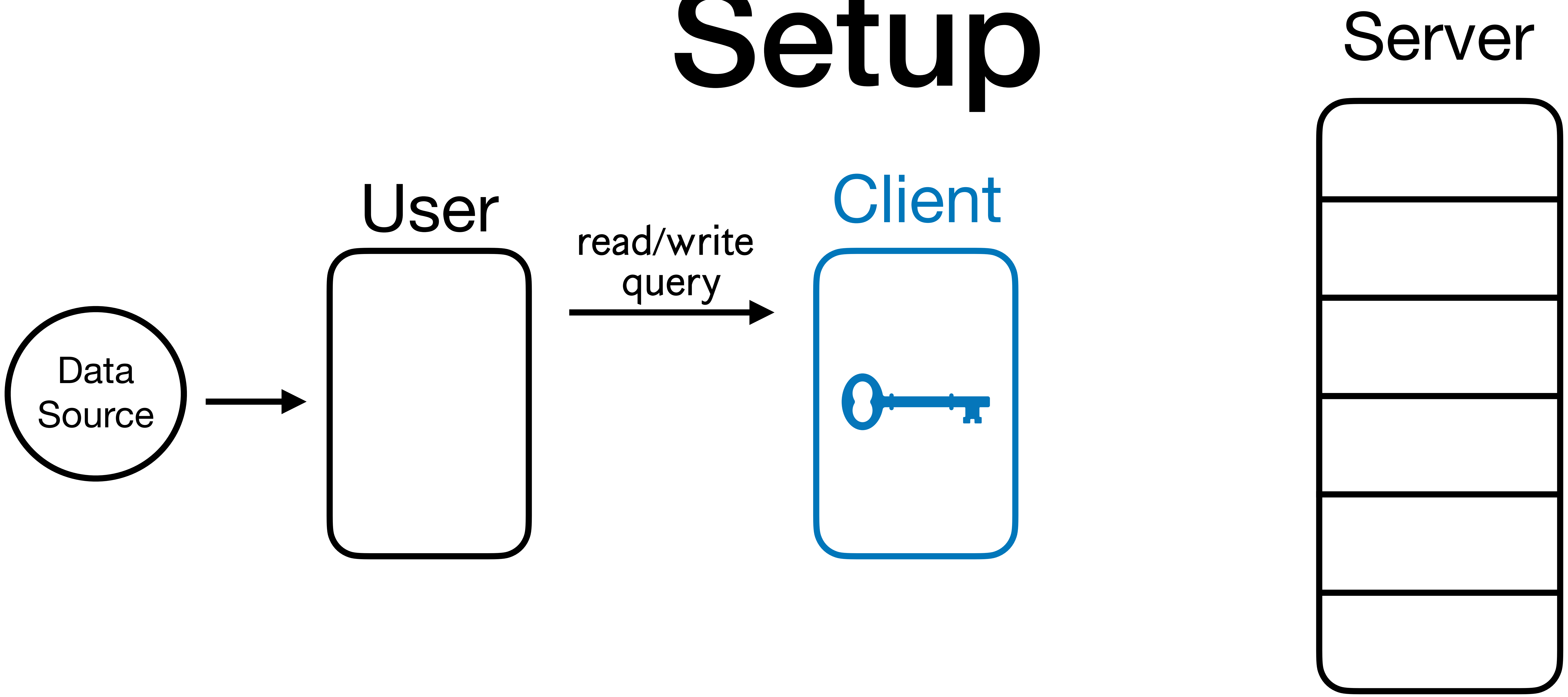

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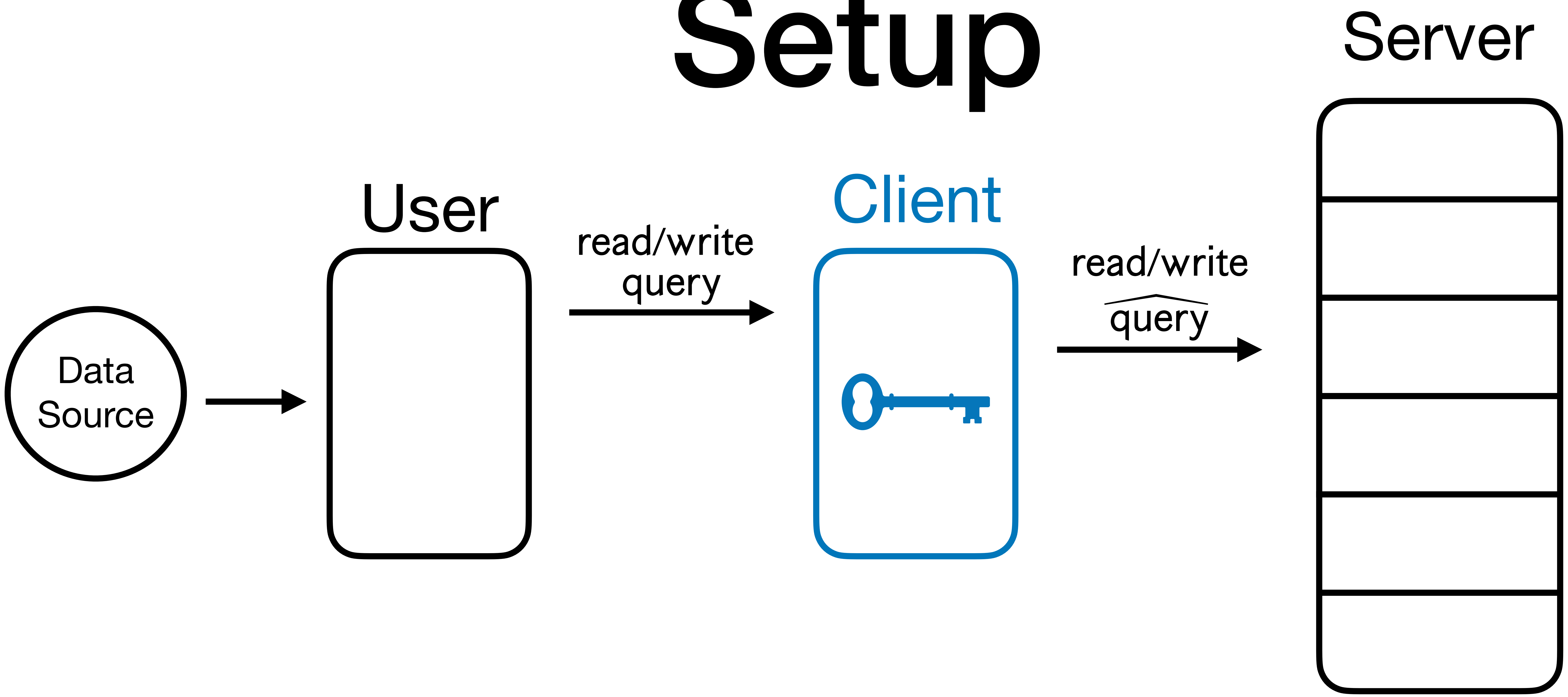
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read/write query
Client
read/write
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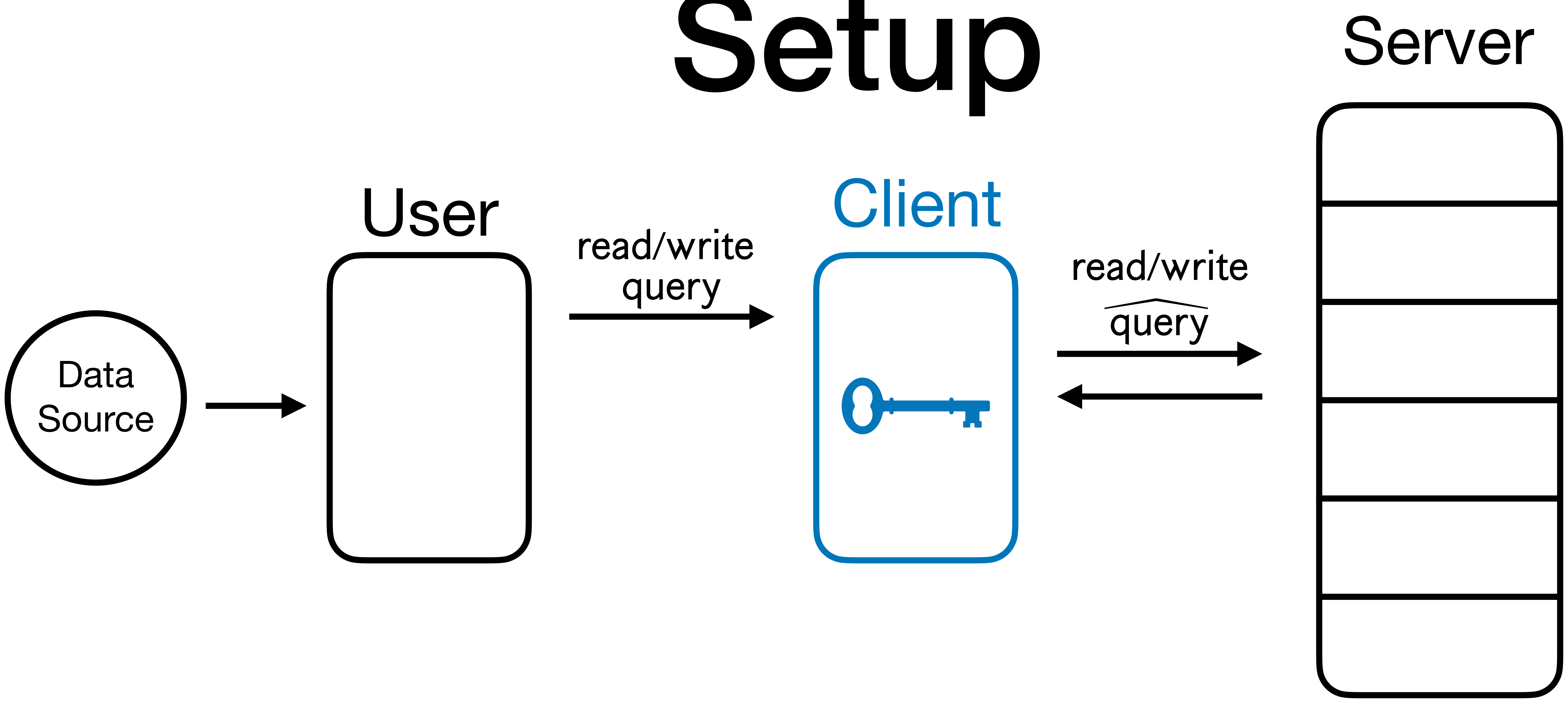
Server

Setup



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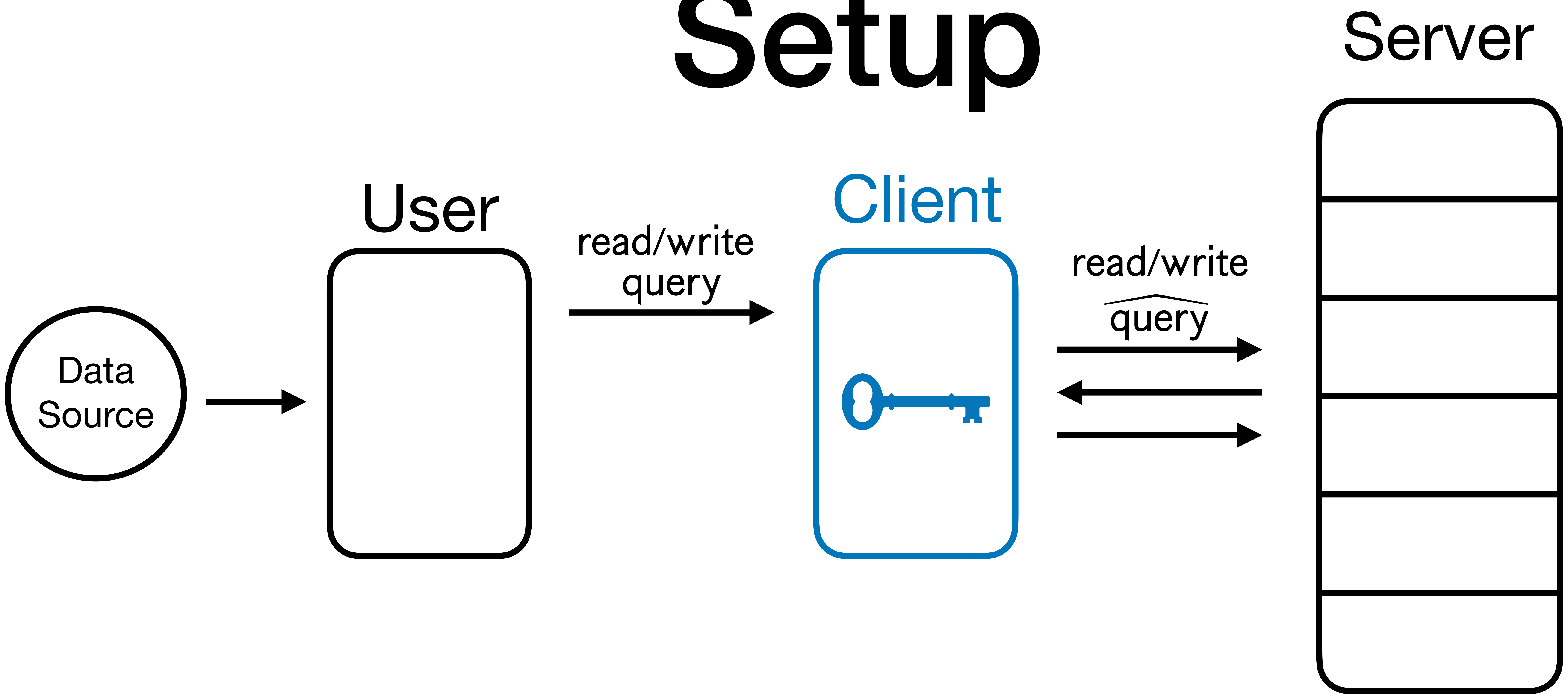
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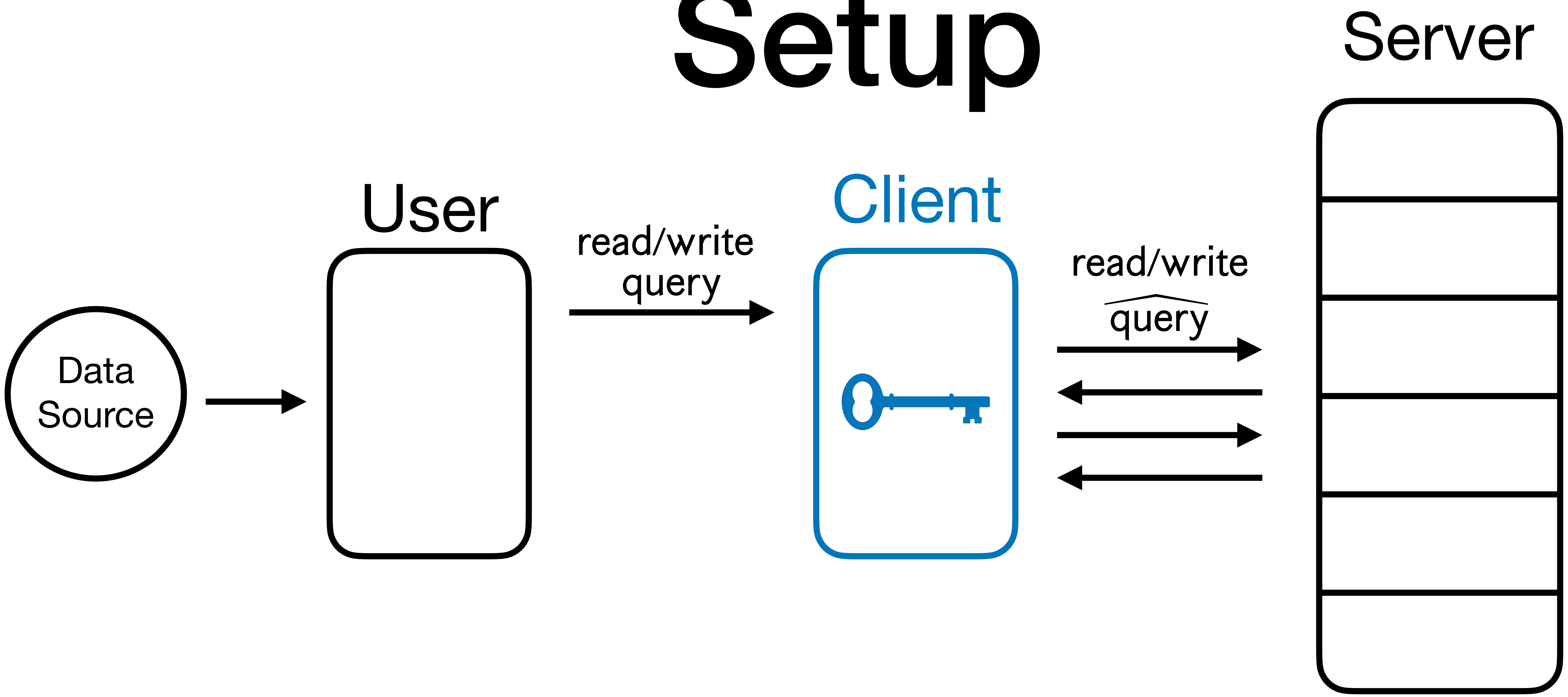
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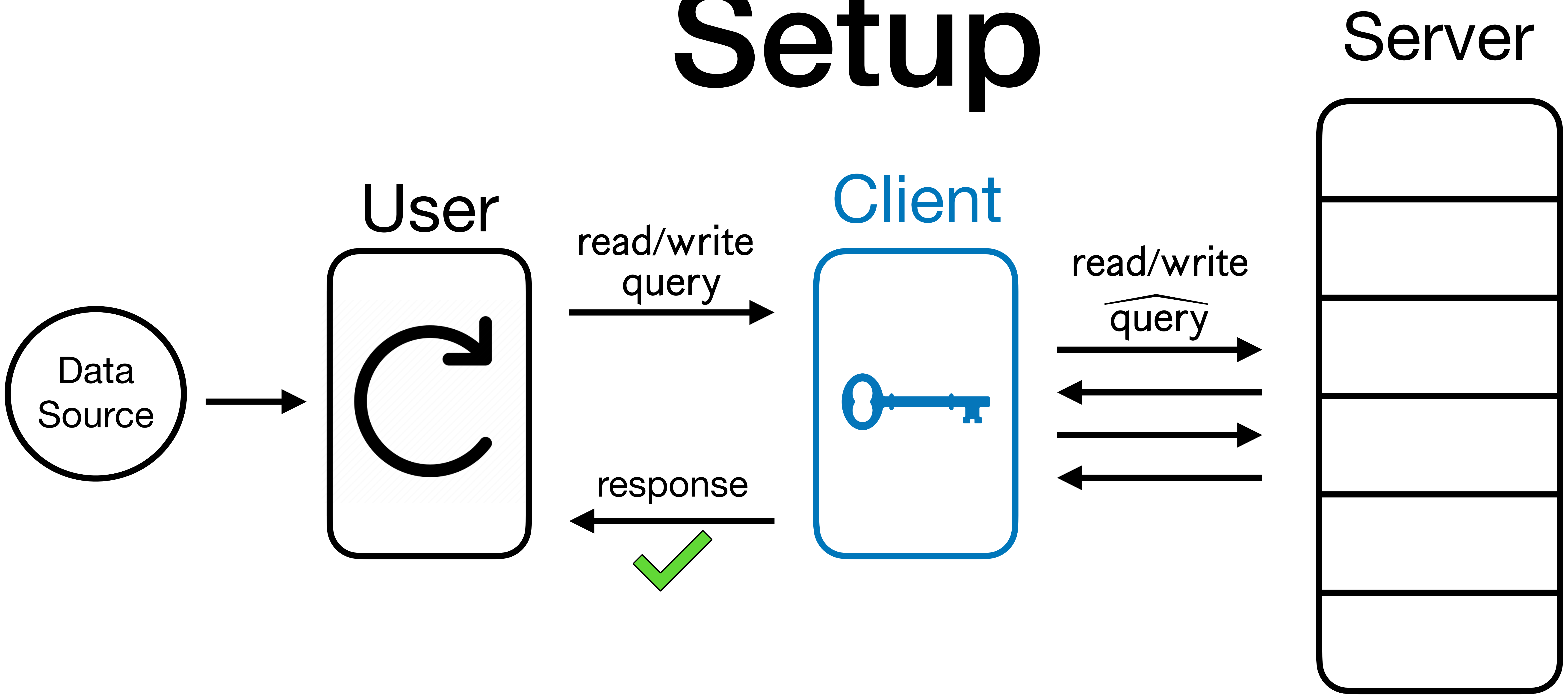
Server
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response

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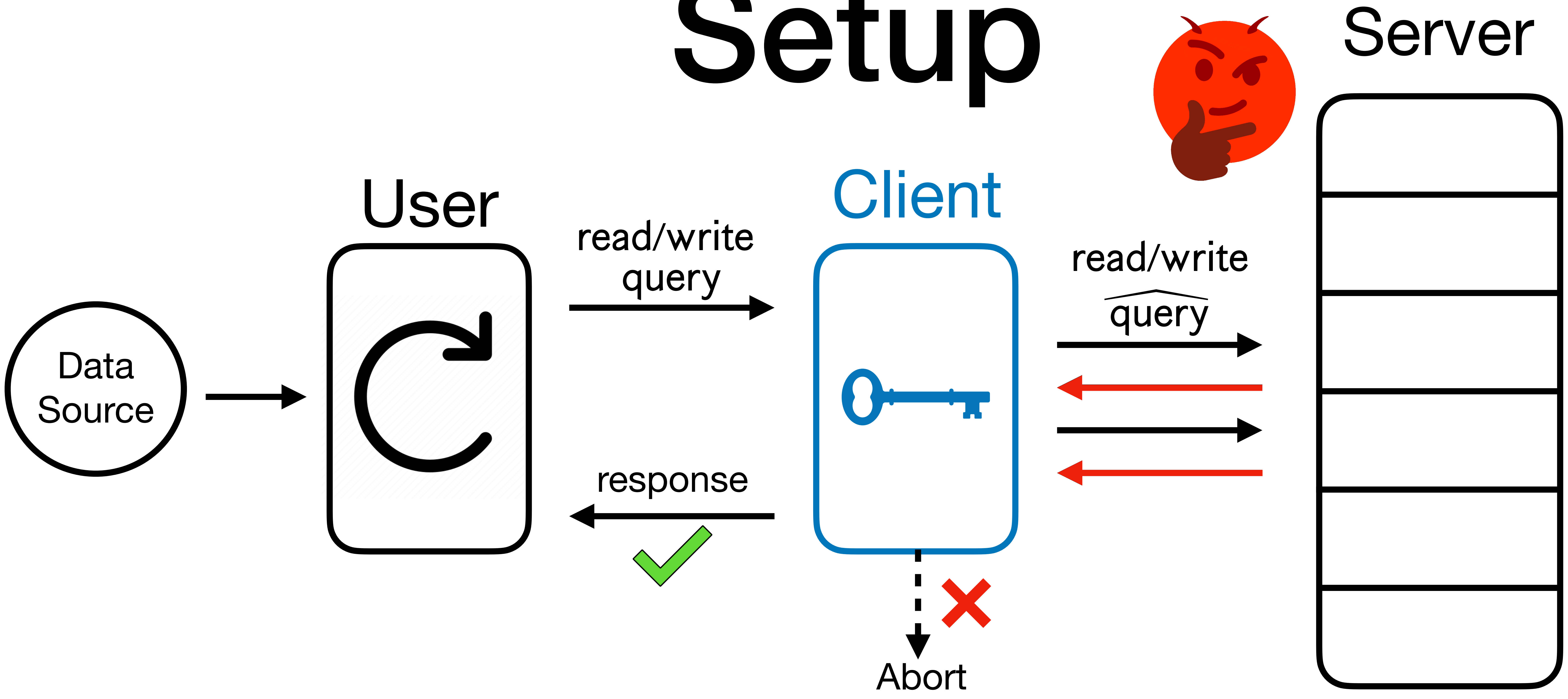

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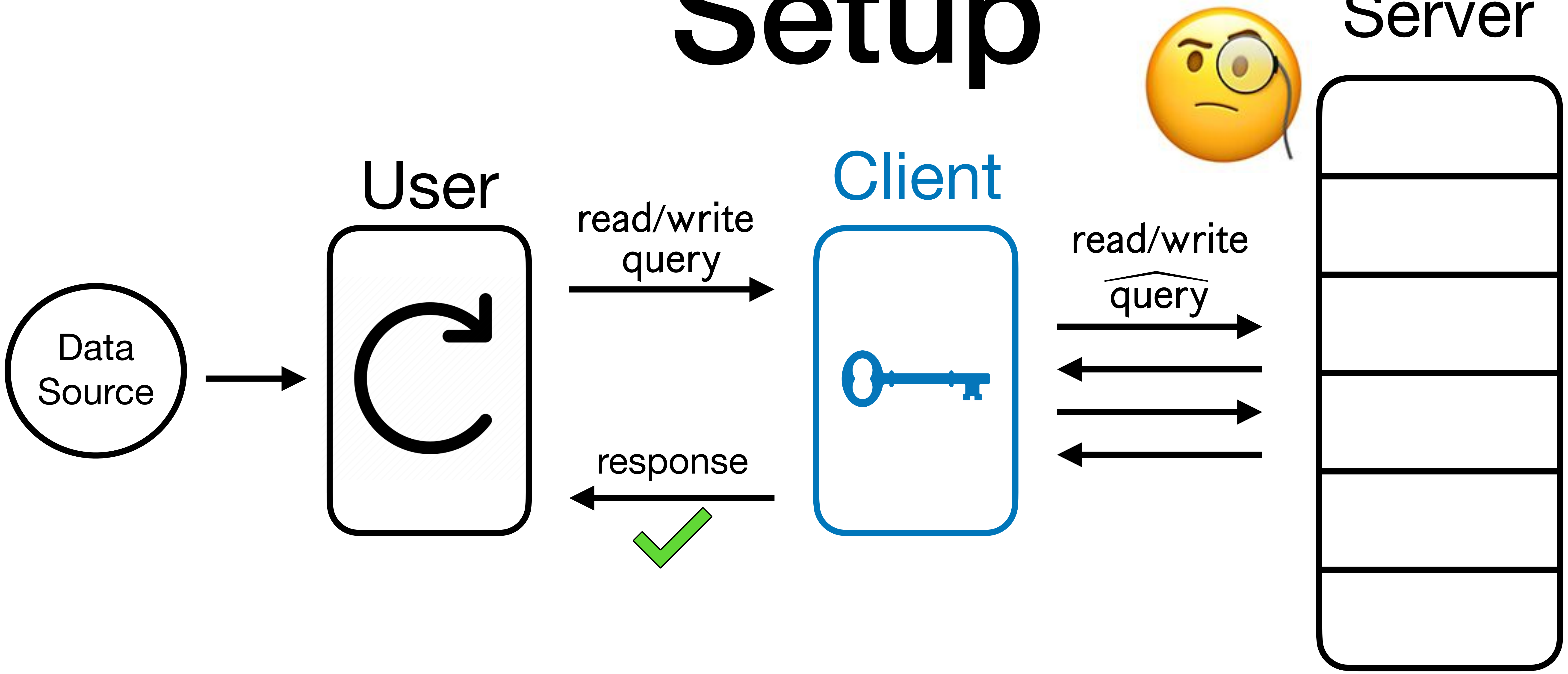

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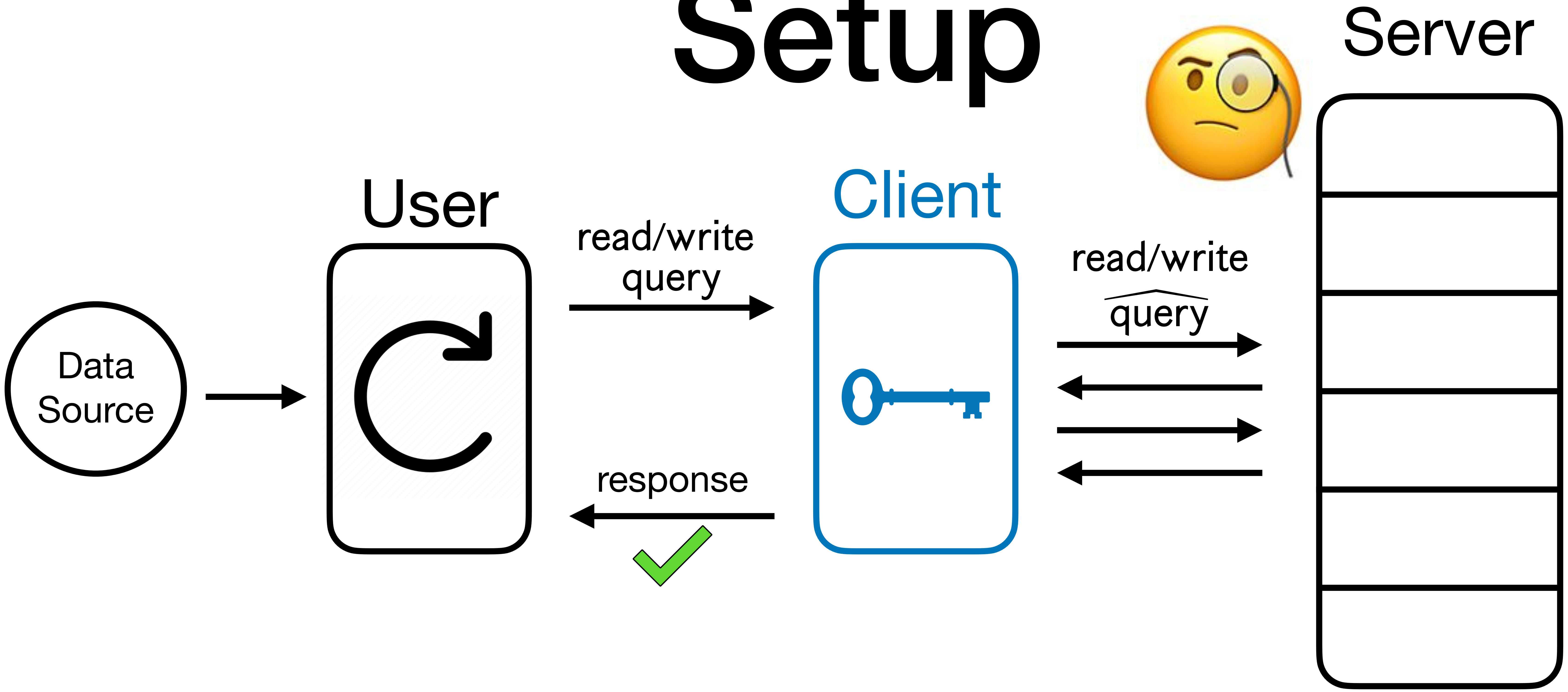

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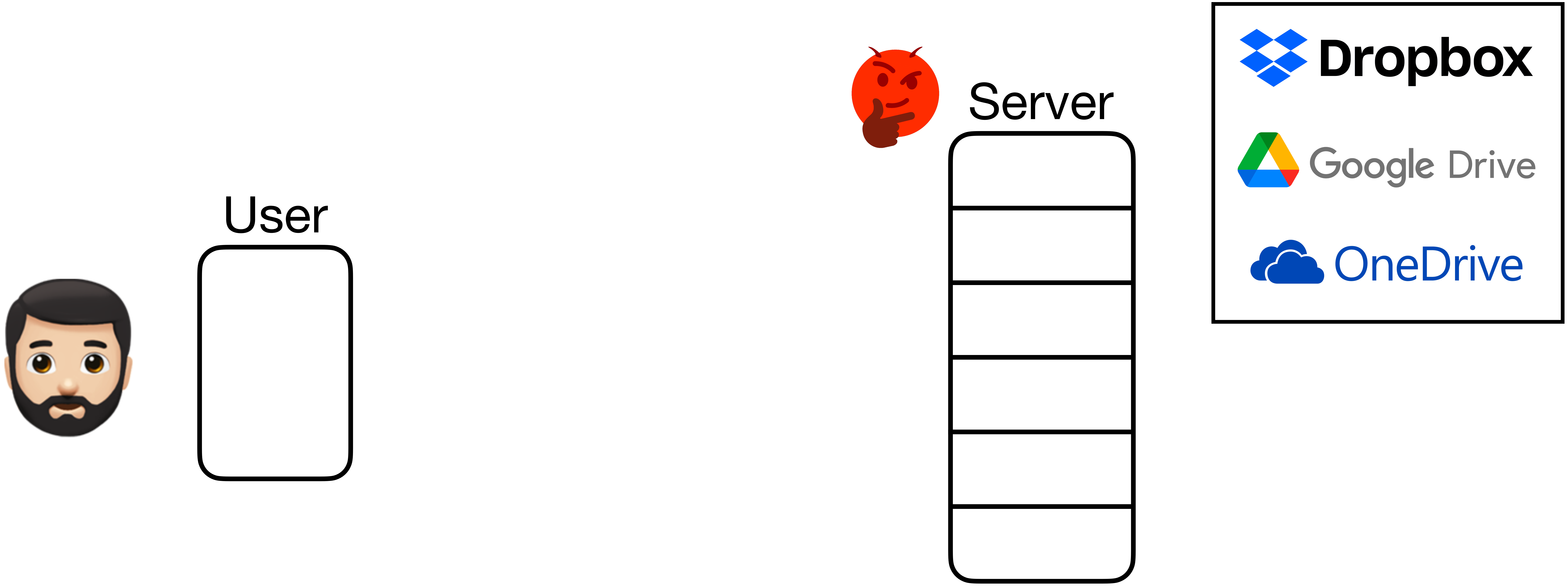

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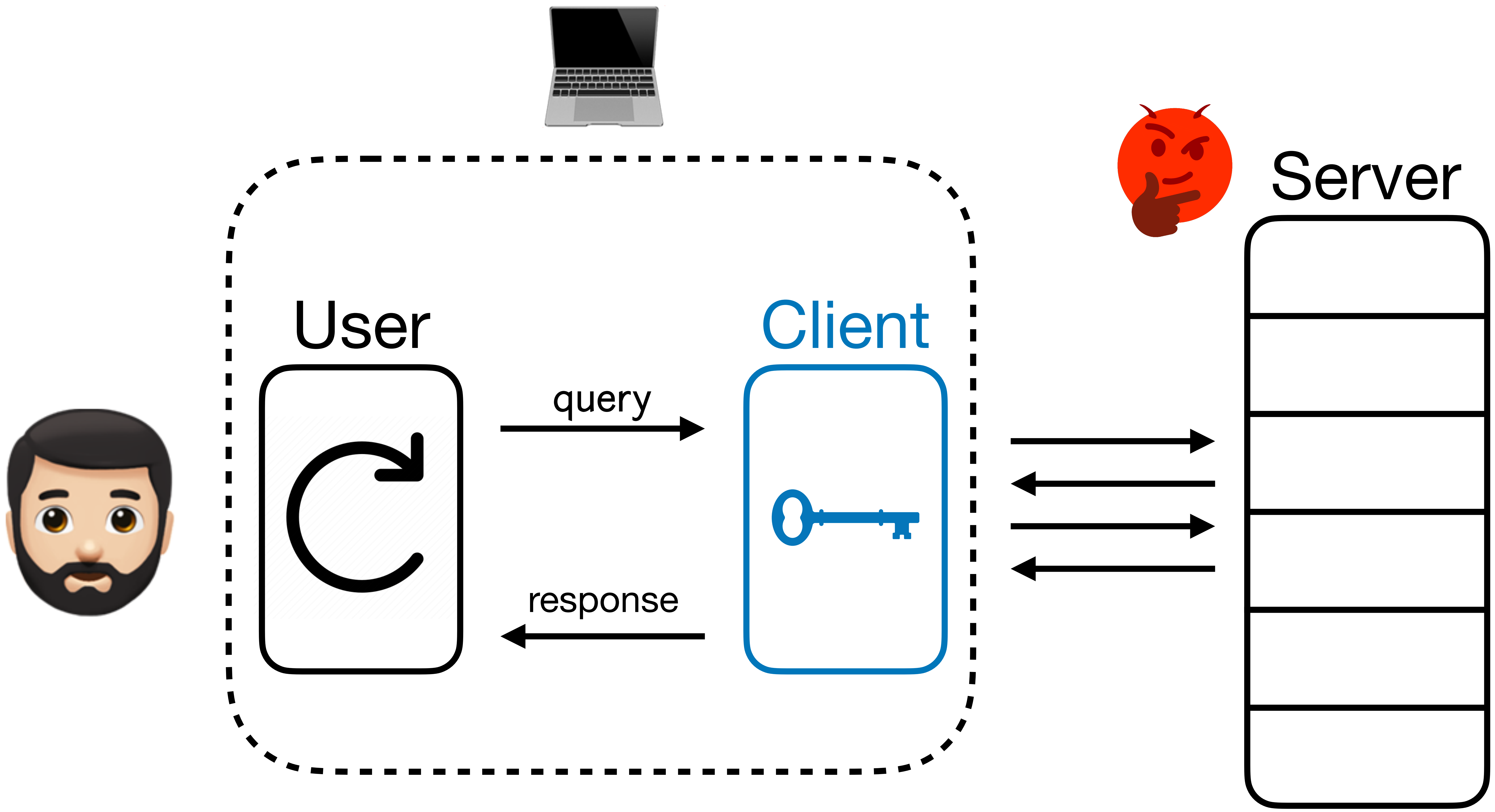

Dropbox

Google Drive

OneDrive

Application: Secure Hardware Enclaves

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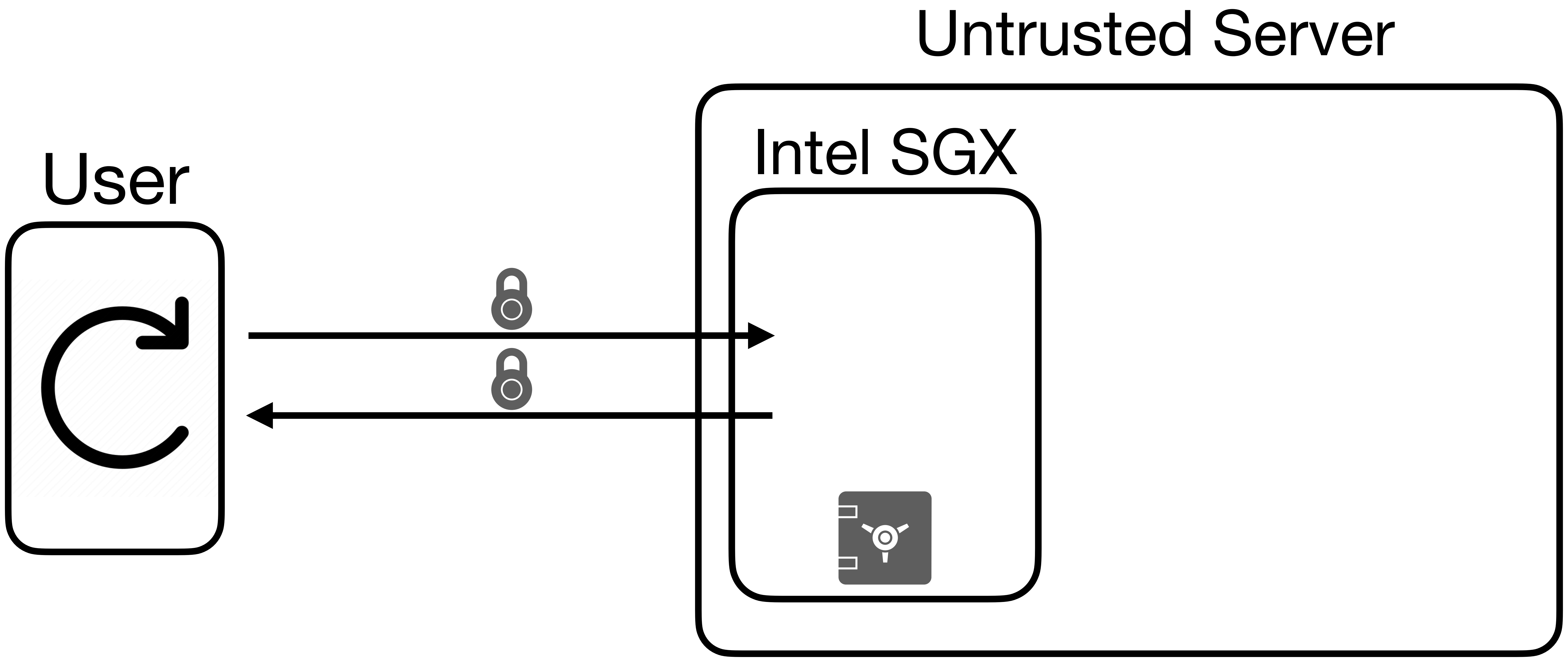

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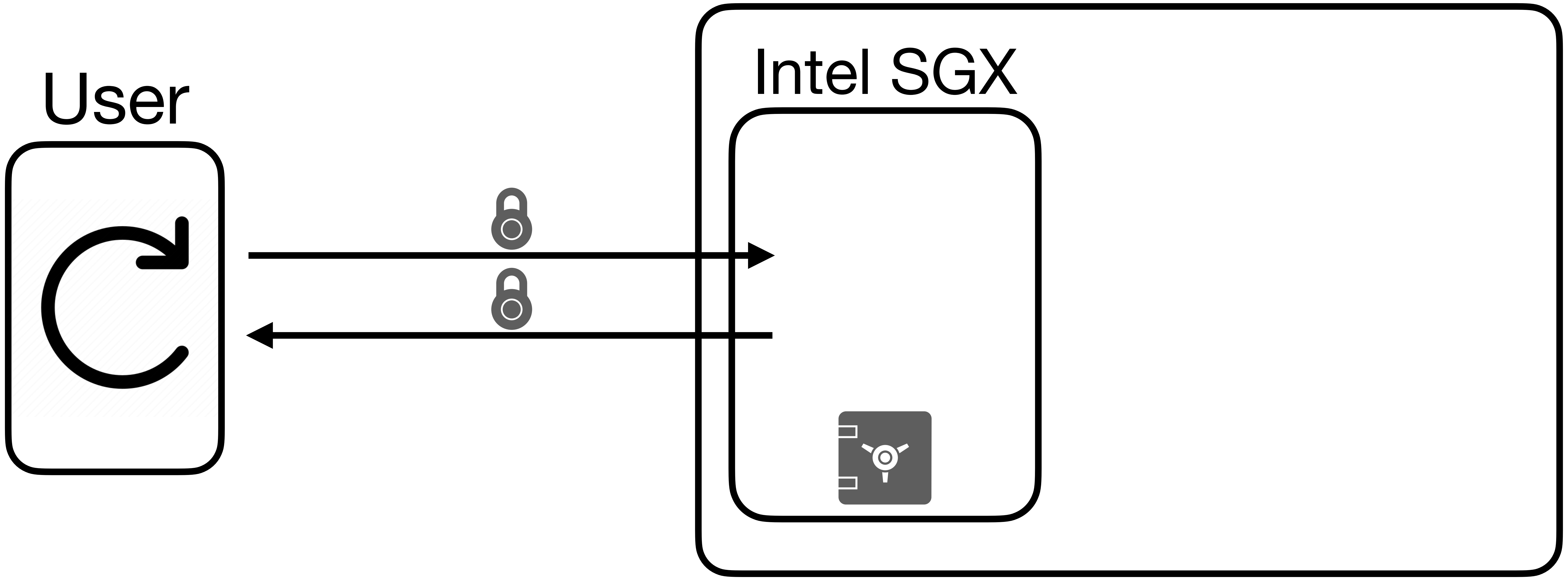

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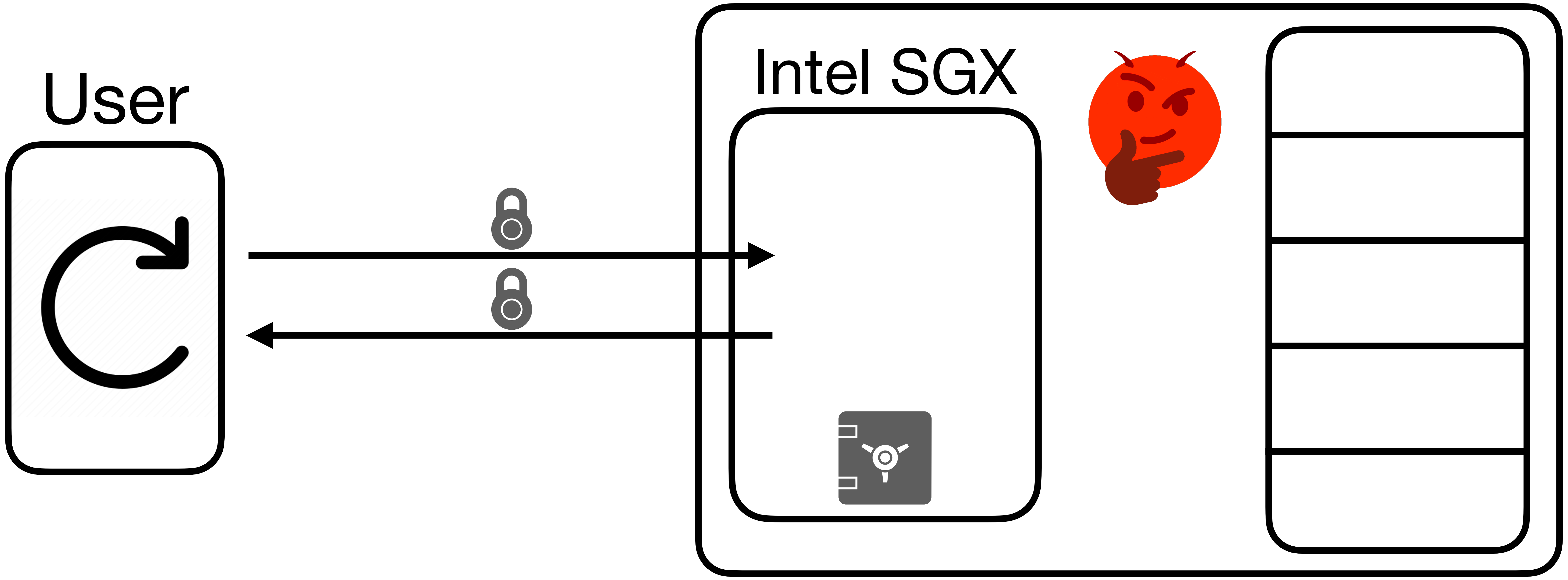

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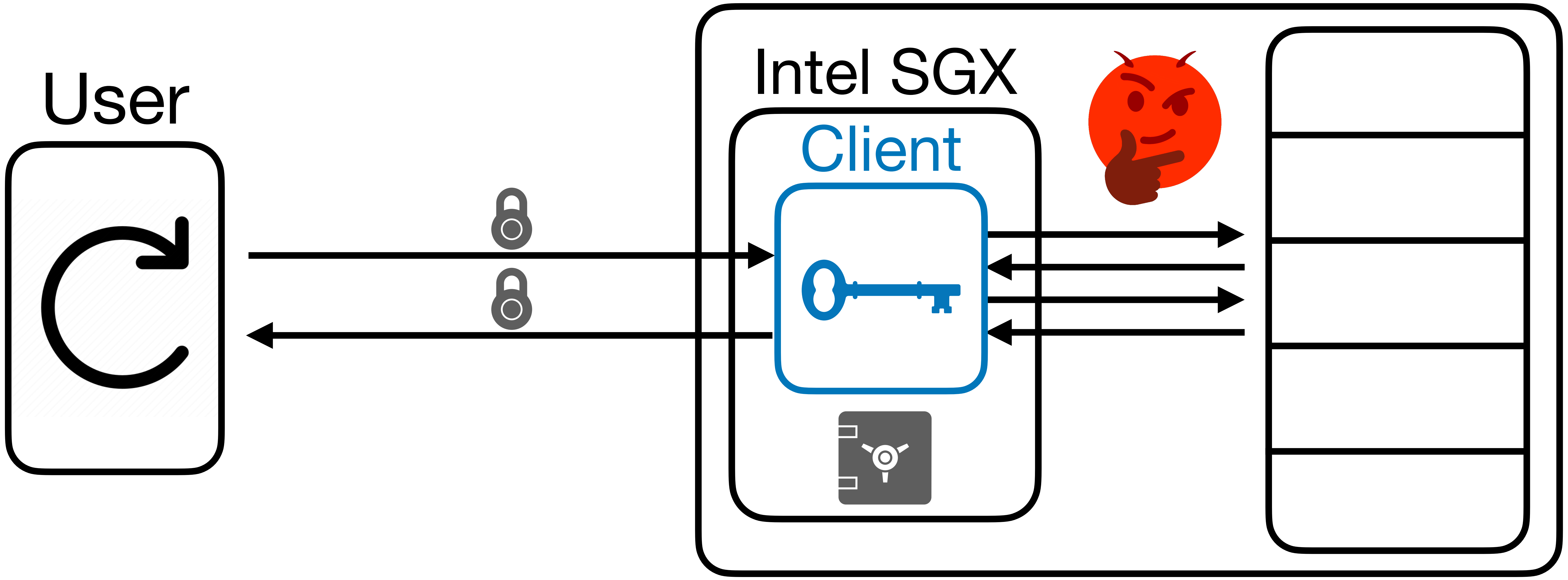

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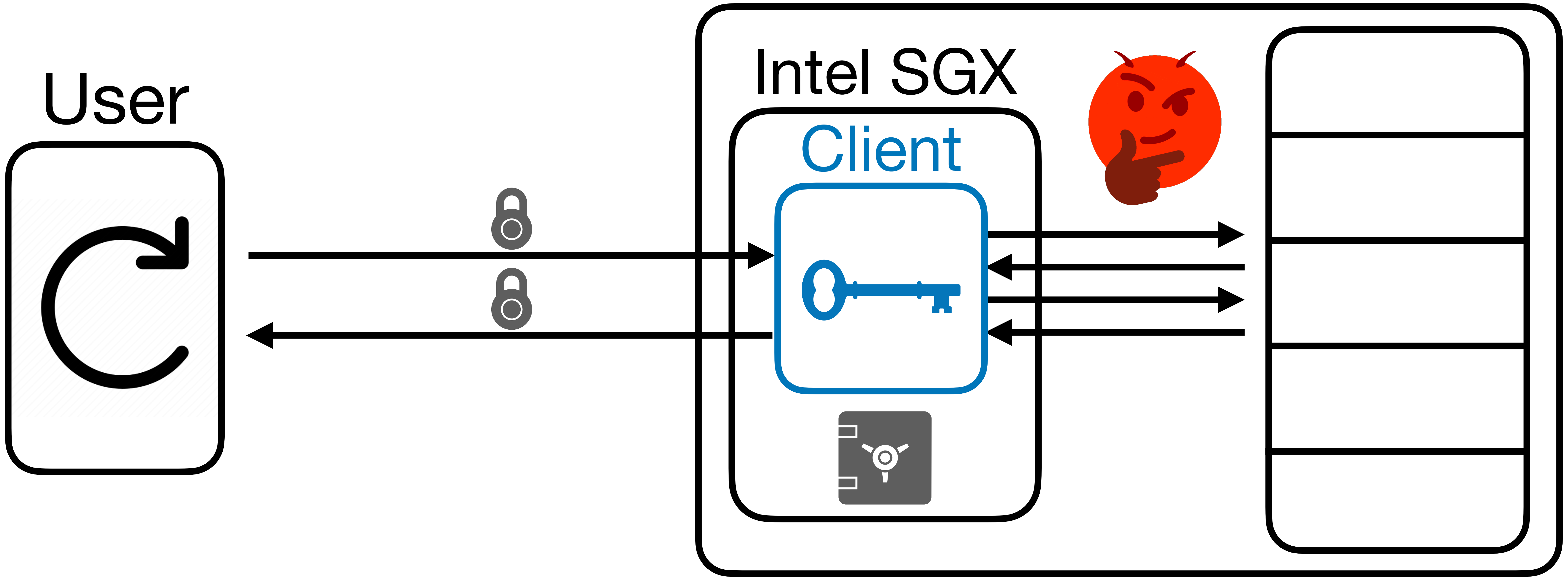

- **Real World**: Signal very recently implemented ORAM for private contact discovery!

Signal

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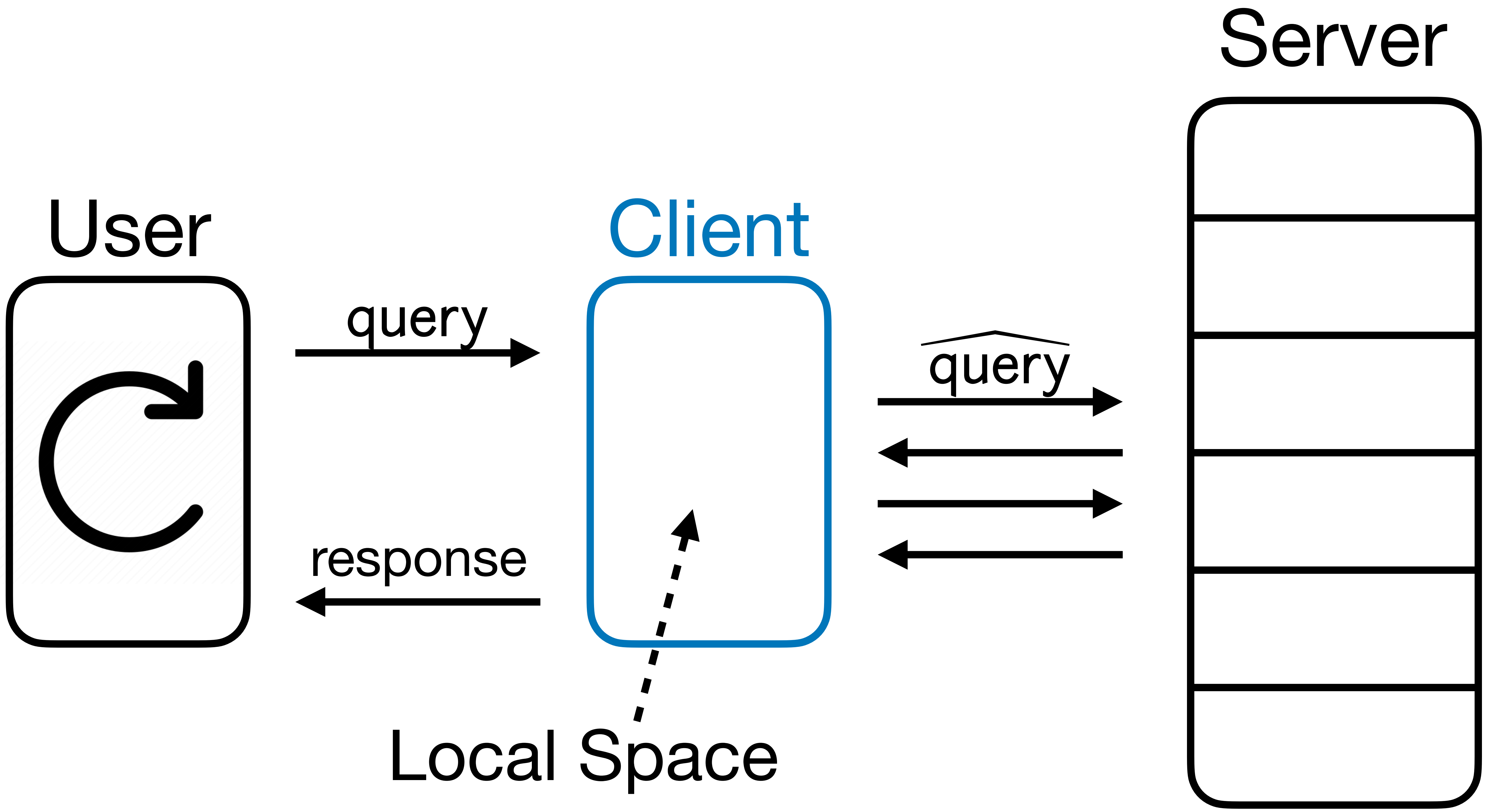
   - For the rest of lecture, think space N^{ϵ} or $\mathsf{polylog}(N)$.

Efficiency


Server
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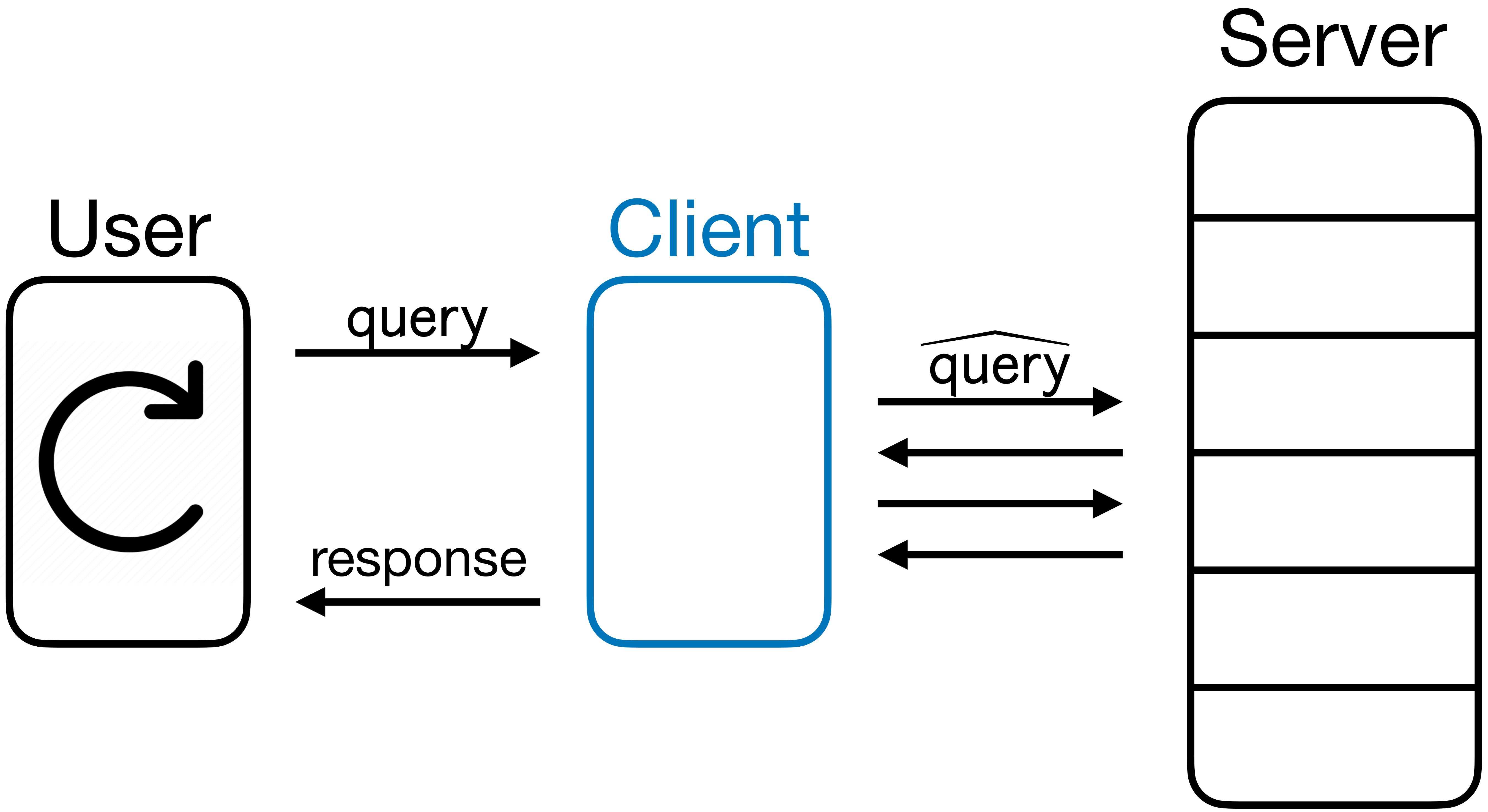
Local Space

Efficiency



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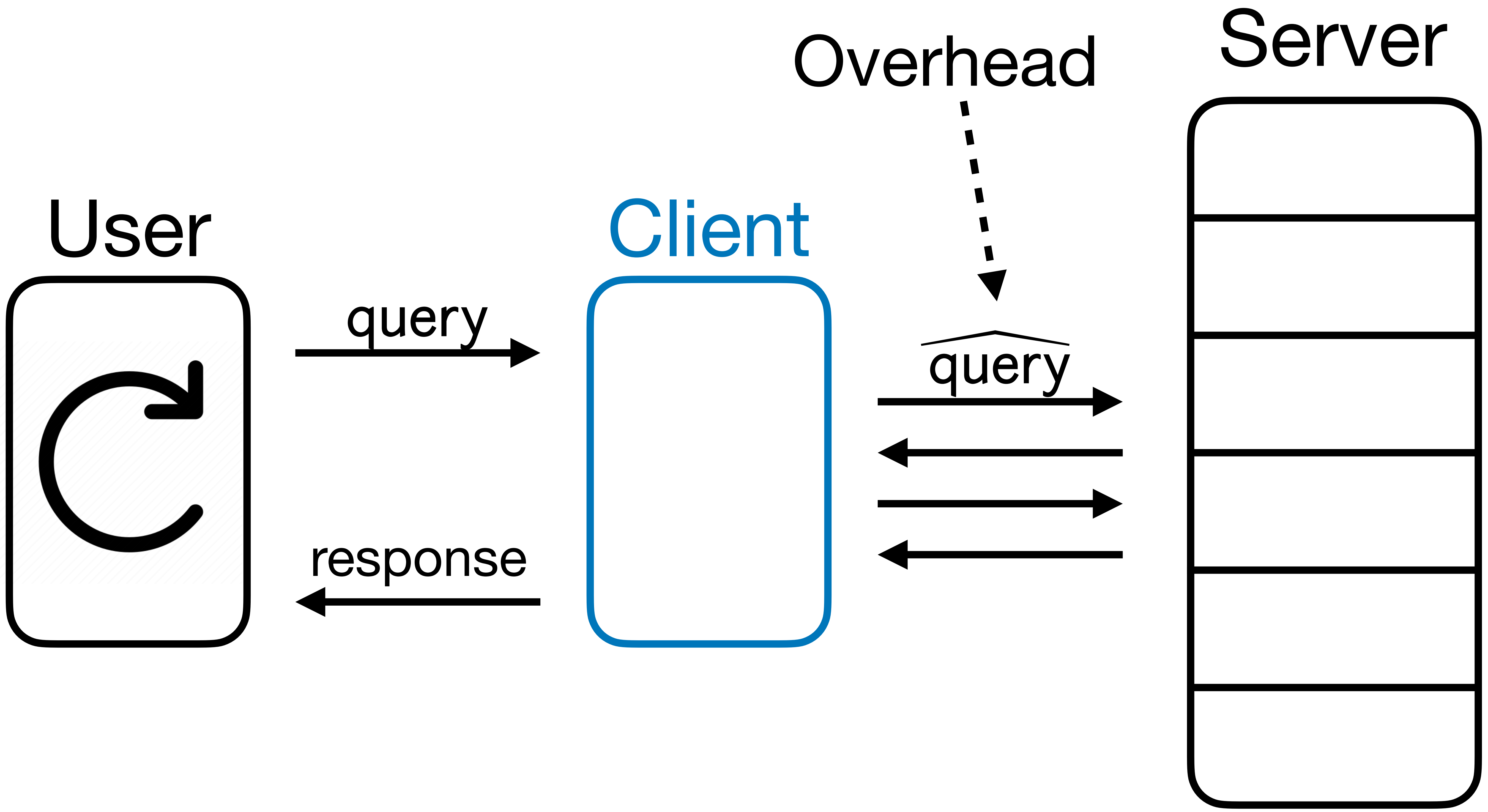

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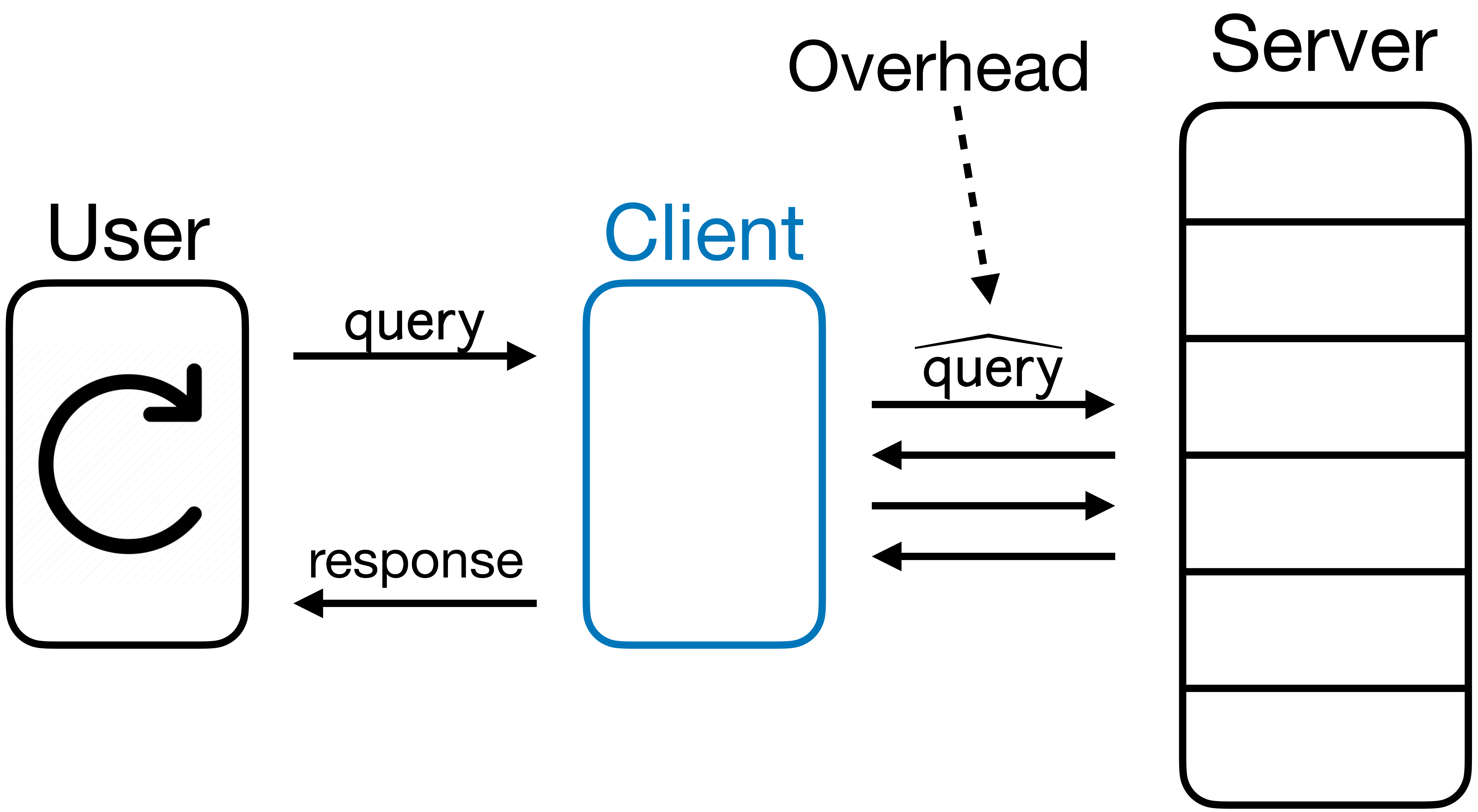

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- We want this to be as small as possible!

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Path ORAM
[SvDSHCFRYD '12]

Memory Checking

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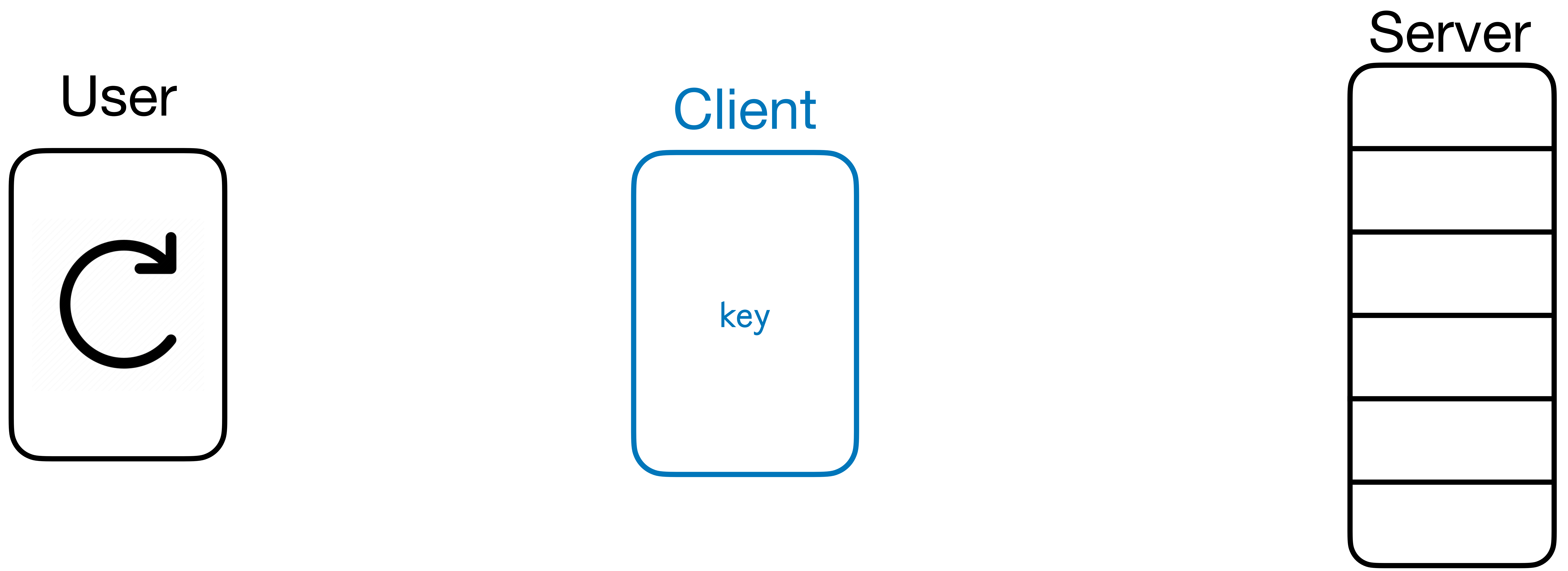
Wait, does authentication solve the integrity issue?
(e.g., MACs, digital signatures)

MACs for Memory Checking?



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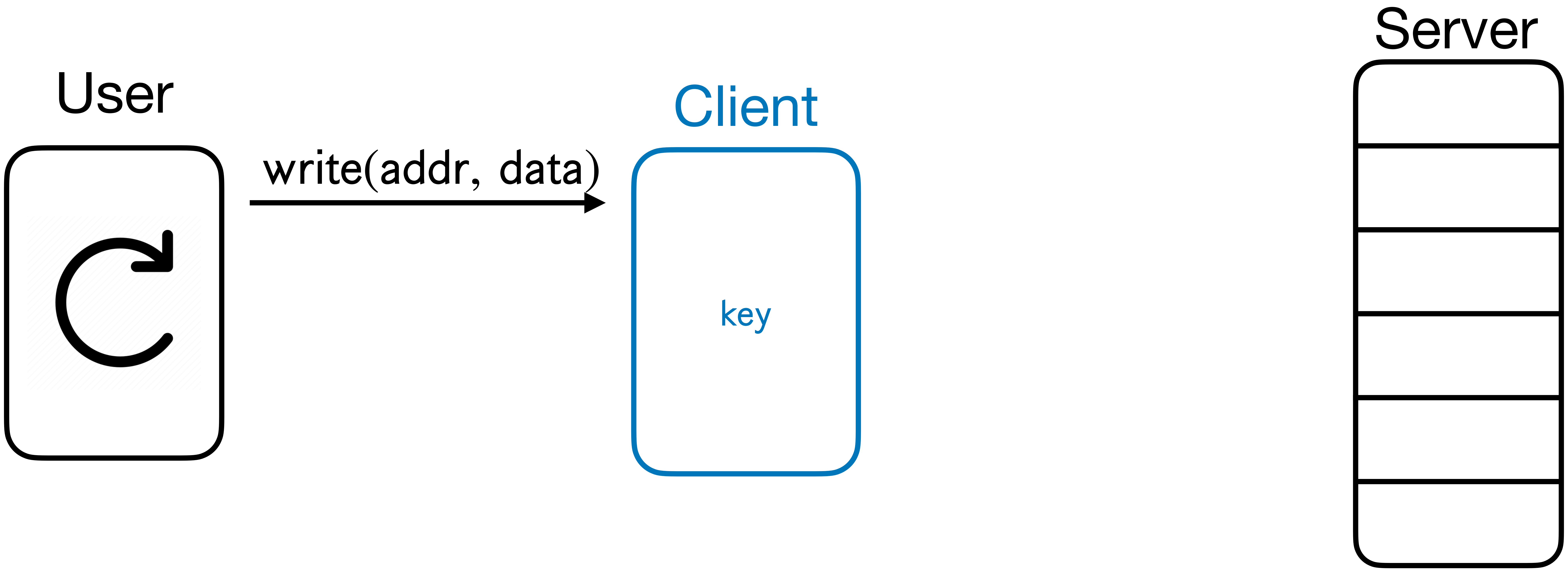

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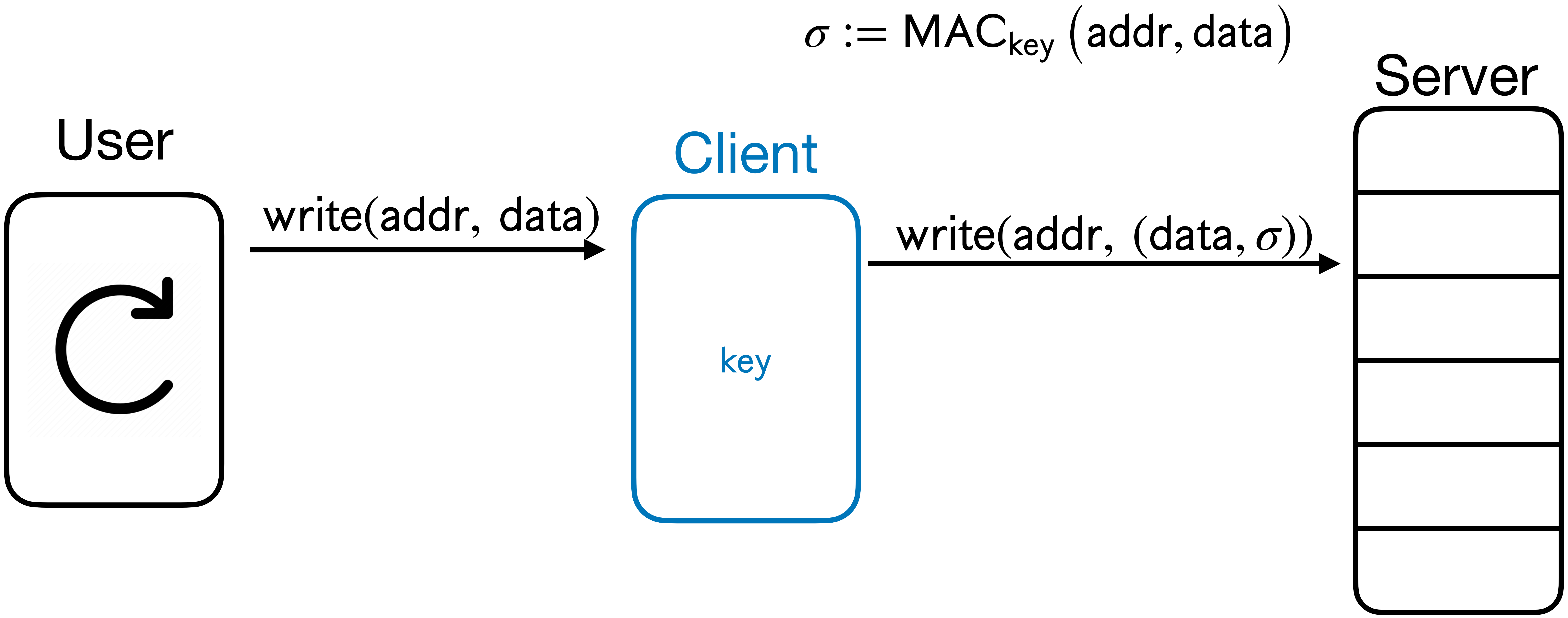

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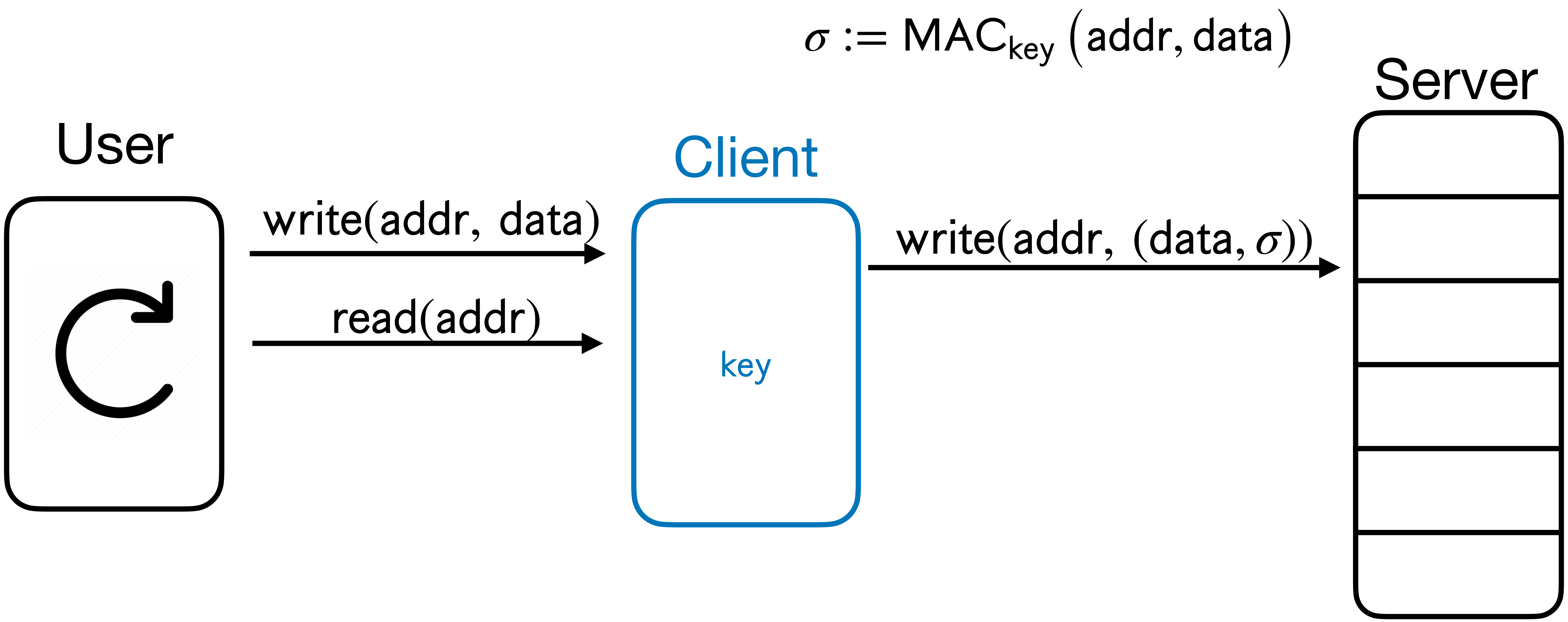

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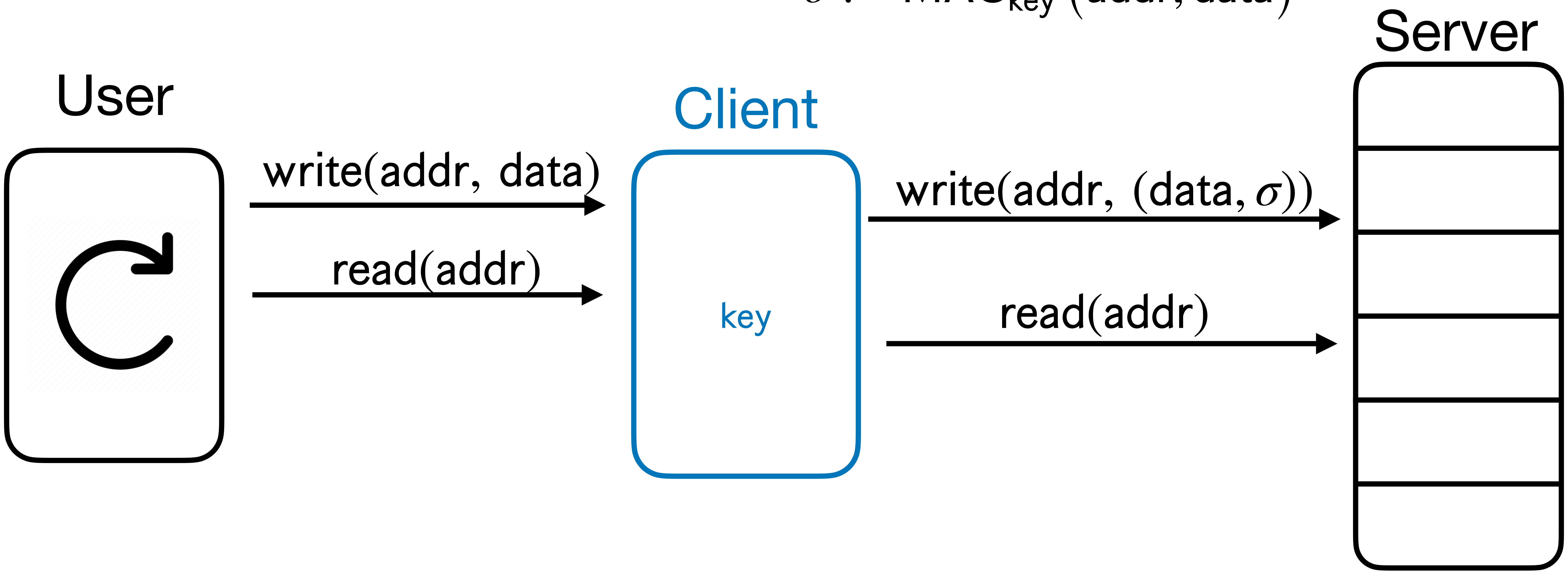
$\sigma := \mathrm{MAC}_{\mathrm{key}}\left(\mathrm{addr}, \mathrm{data}\right)$



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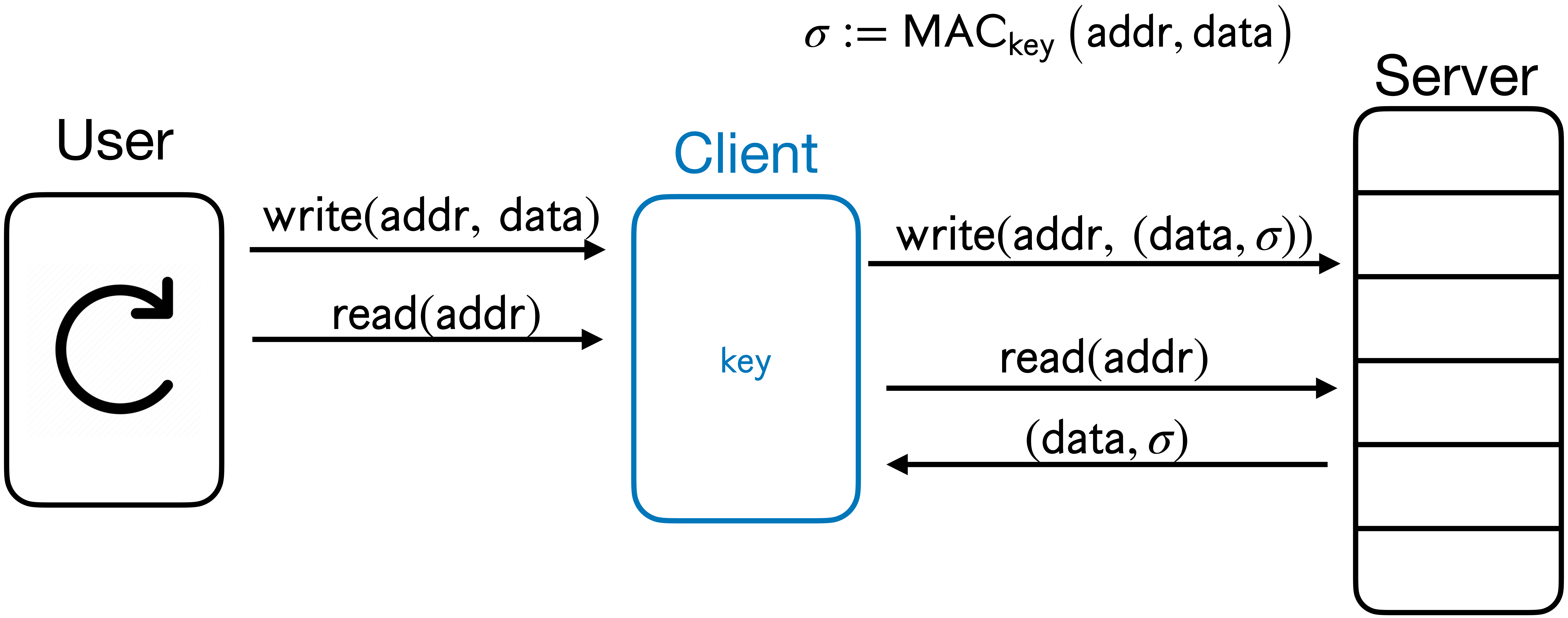

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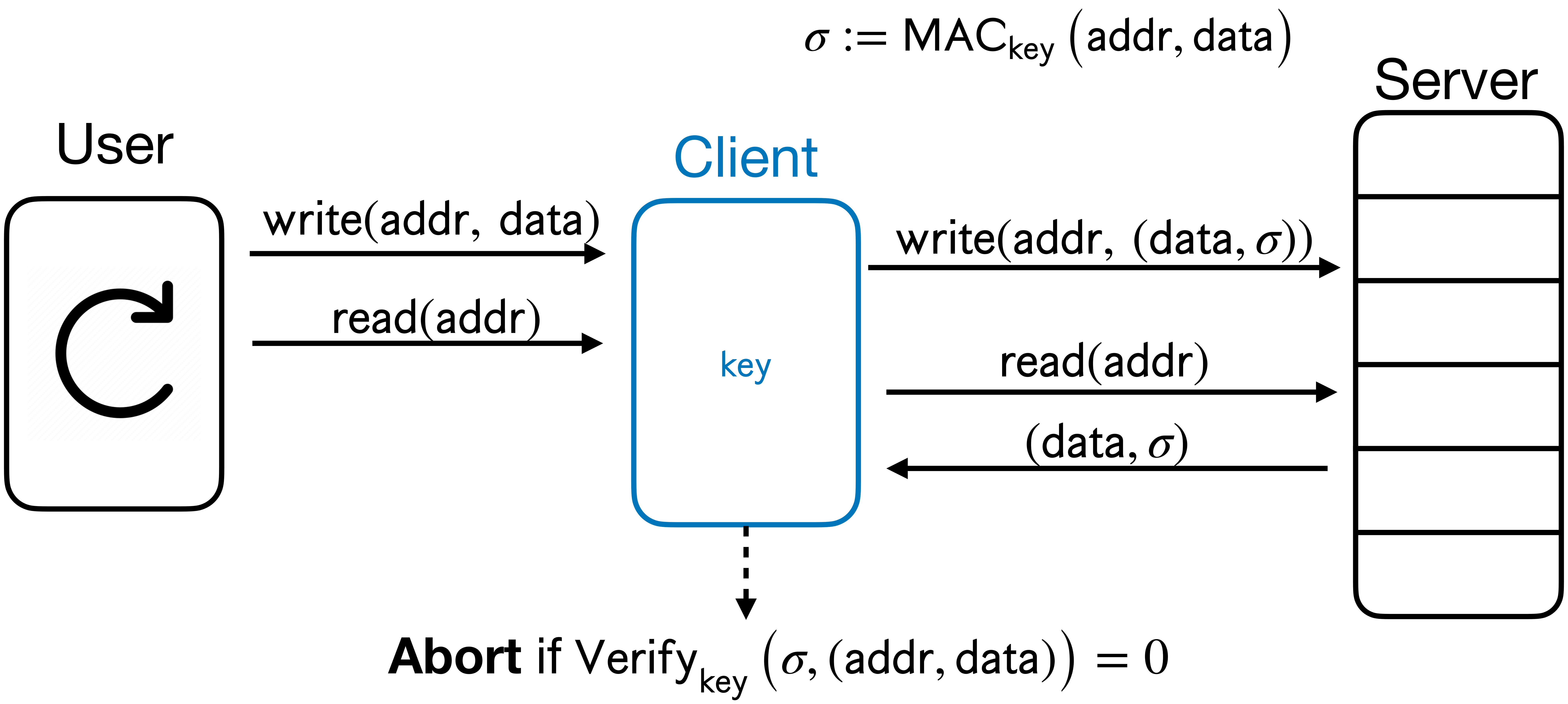

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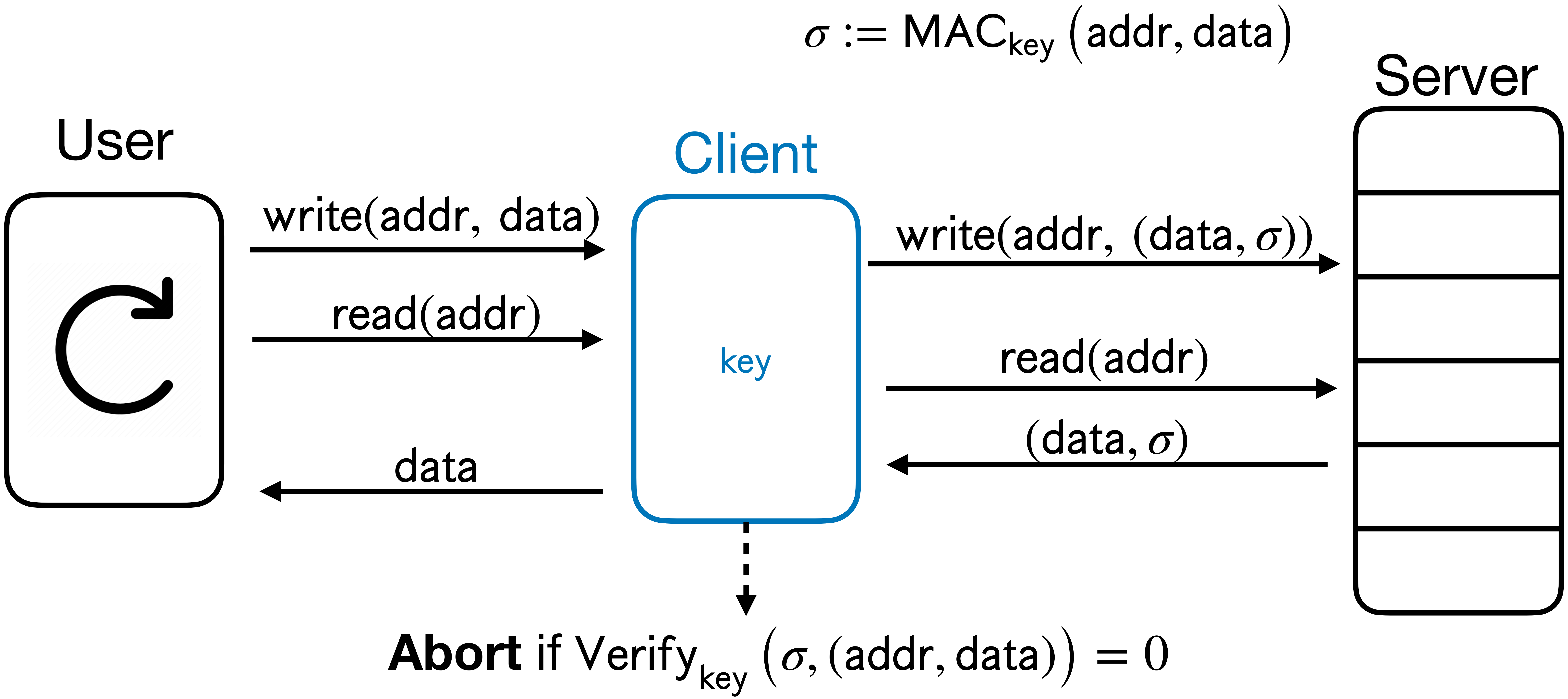

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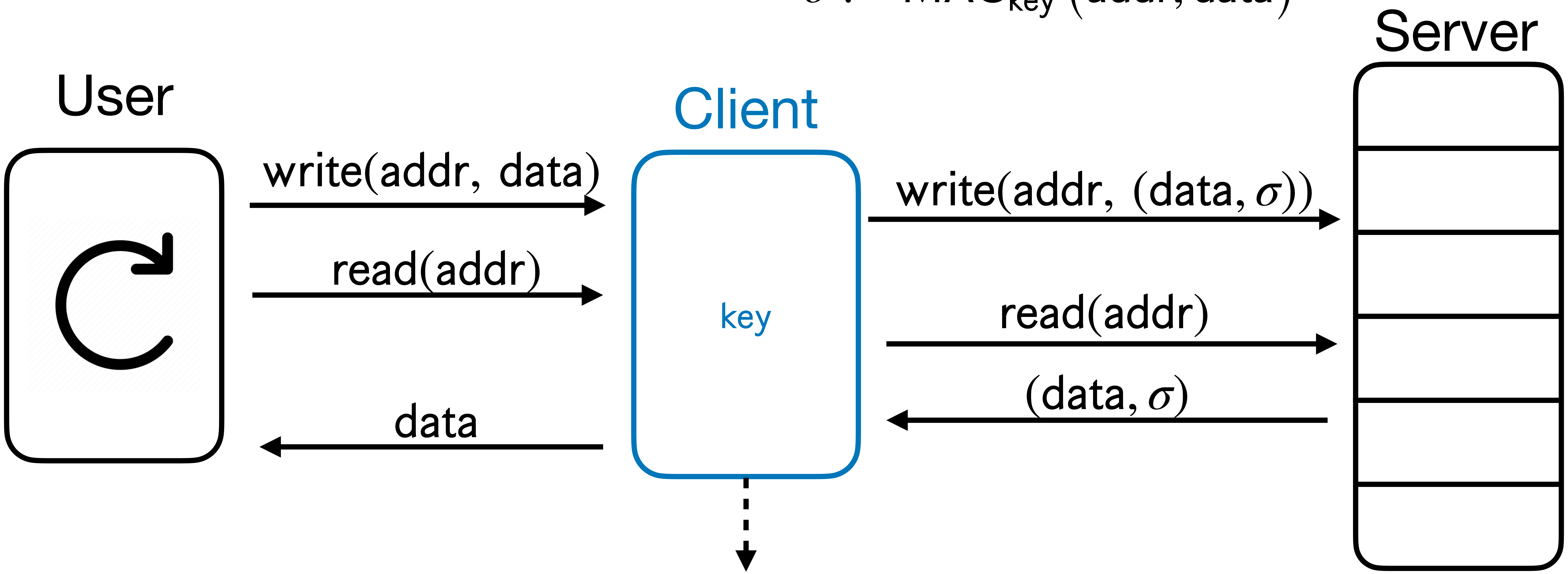

Abort if $\mathsf{Verify}_{\mathsf{key}}(\sigma, (\mathsf{addr}, \mathsf{data})) = 0$

Does this work? What does it prevent?

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- Natural idea: add counters/time-stamps inside MACs.
- (Fatal) issue: No way to check counters/time-stamps in low space.

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- **Natural idea**: Collision-Resistant Hash Functions (CRHFs)
- **Hope**: Store hash locally, and check correctness of the hash.
- Throughout, let $H : \{0,1\}^* \to \{0,1\}^\lambda$ be a CRHF with $\lambda \ll N$.

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Merkle Trees

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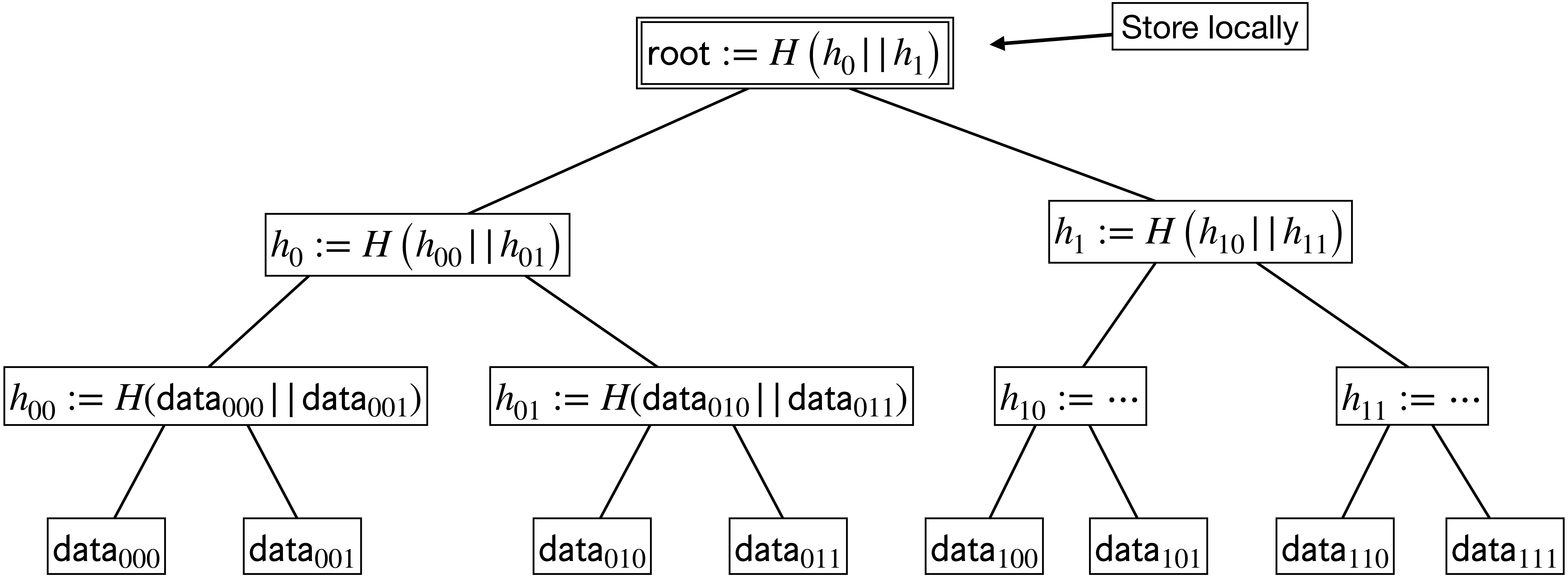
- **Option 3:** Trade off between the two options with a binary tree!

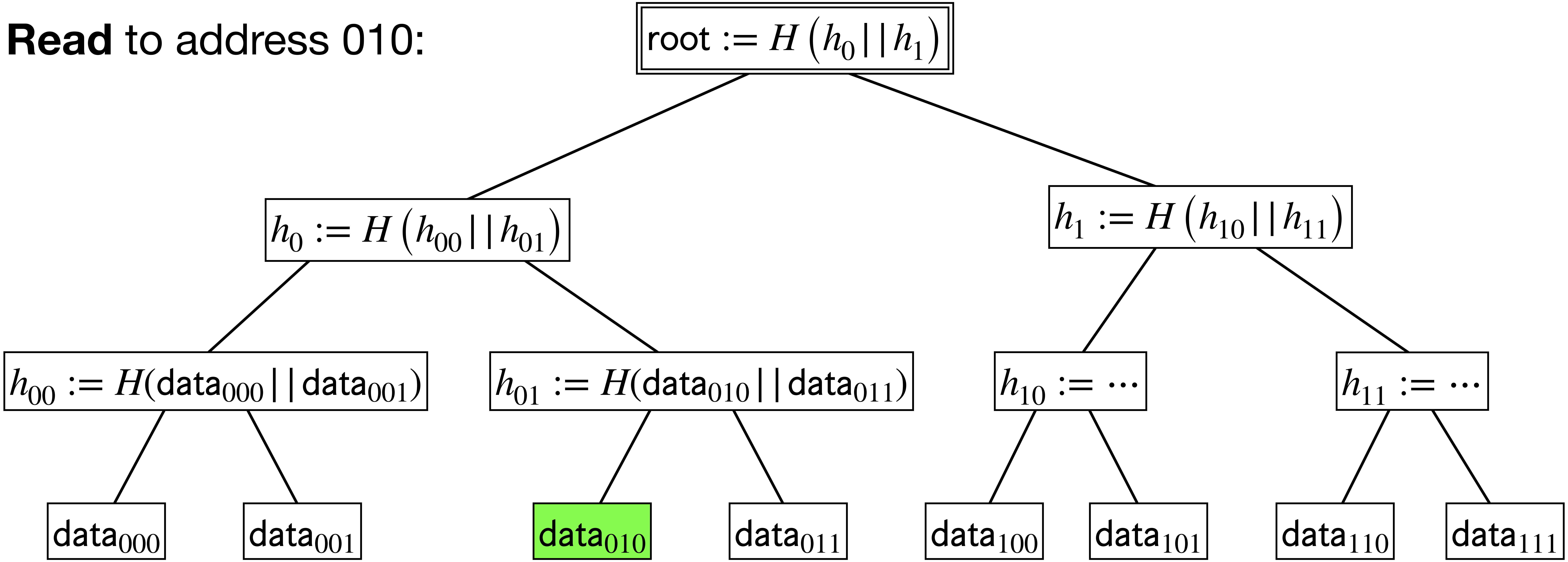
Merkle Trees

(Here, $N = 8$.)



Merkle Trees

Read to address 010:



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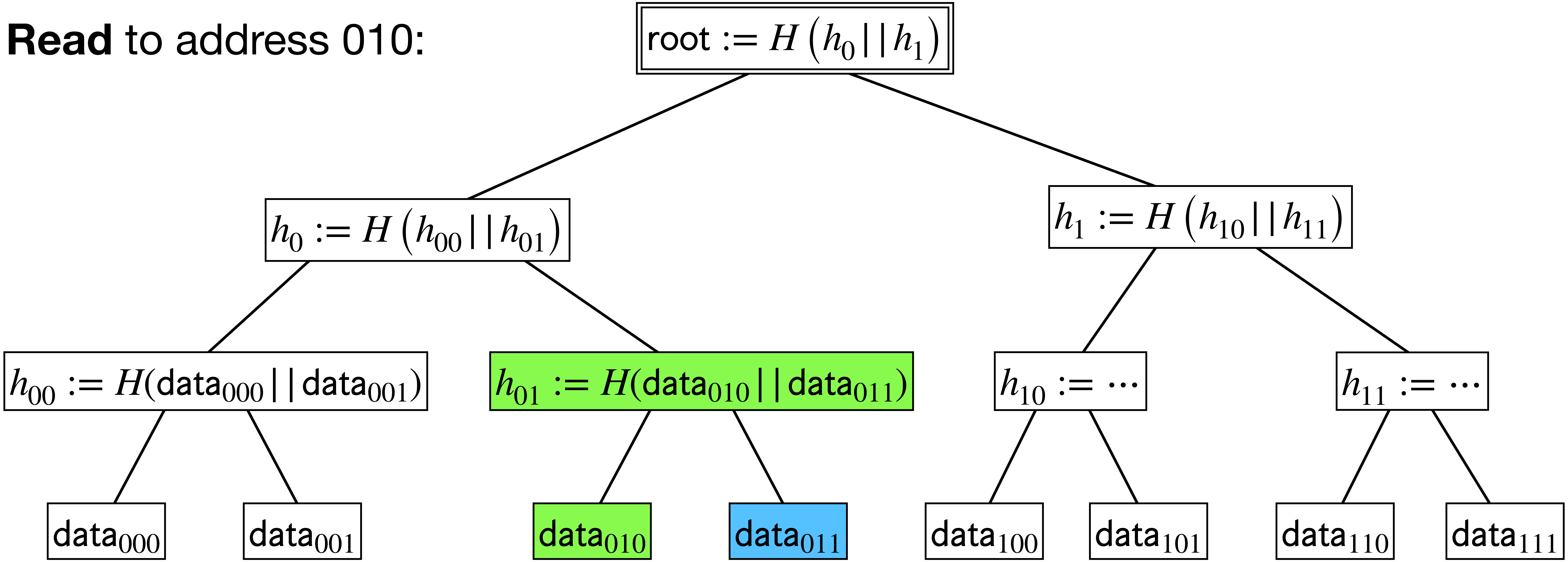

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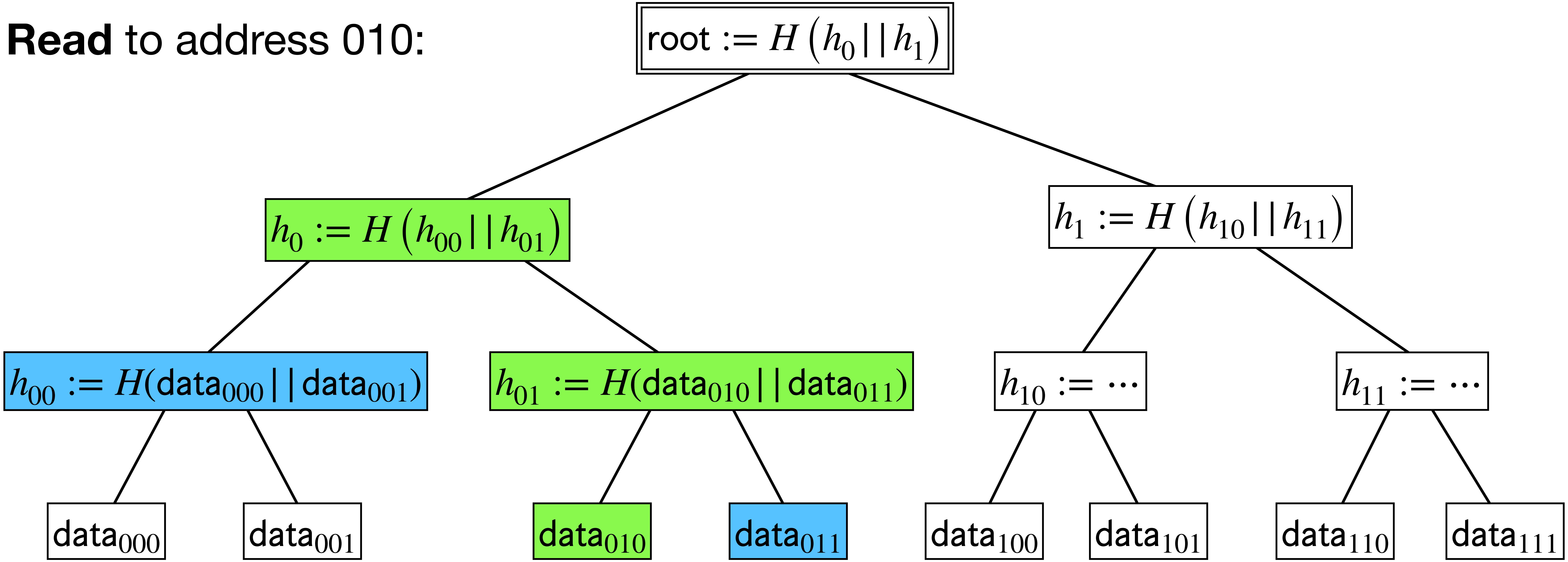

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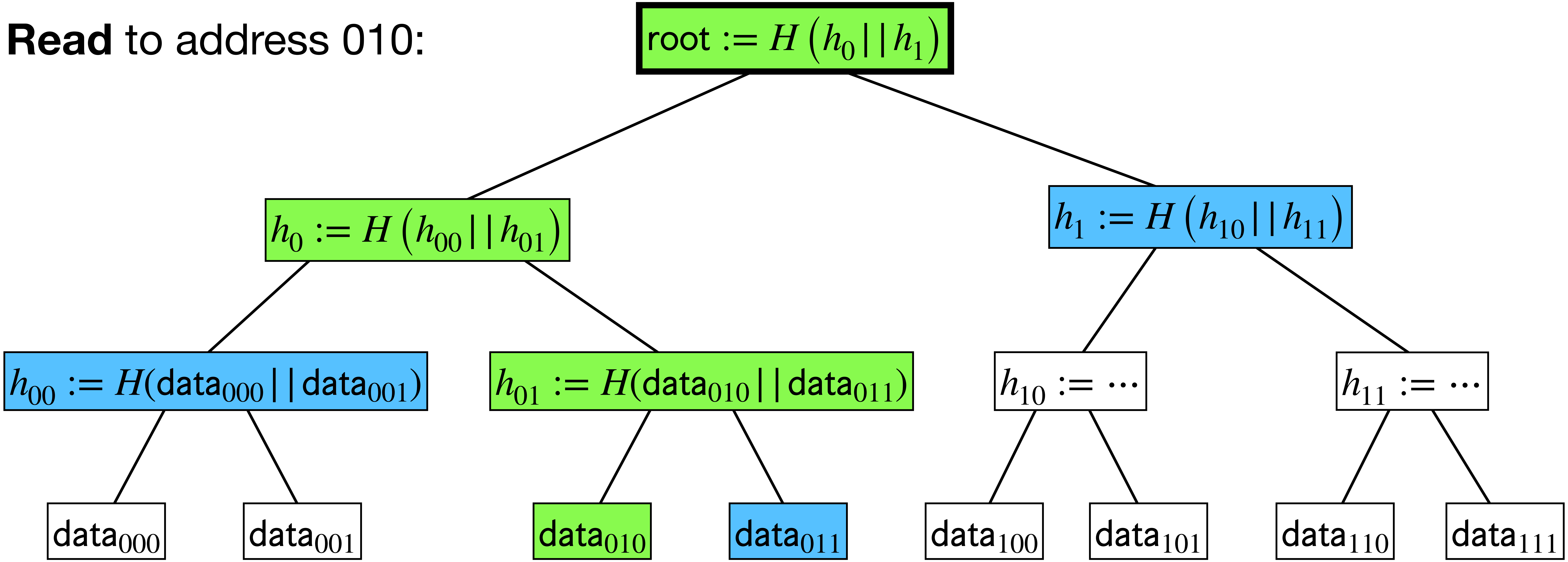

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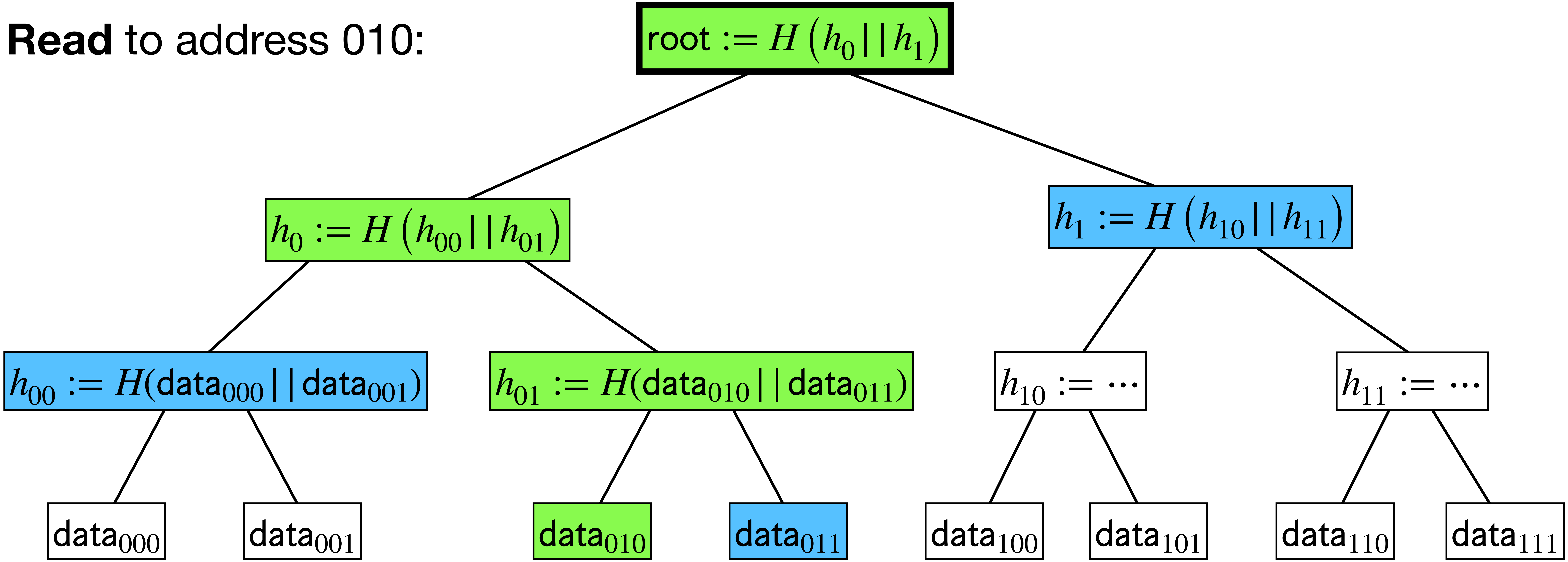
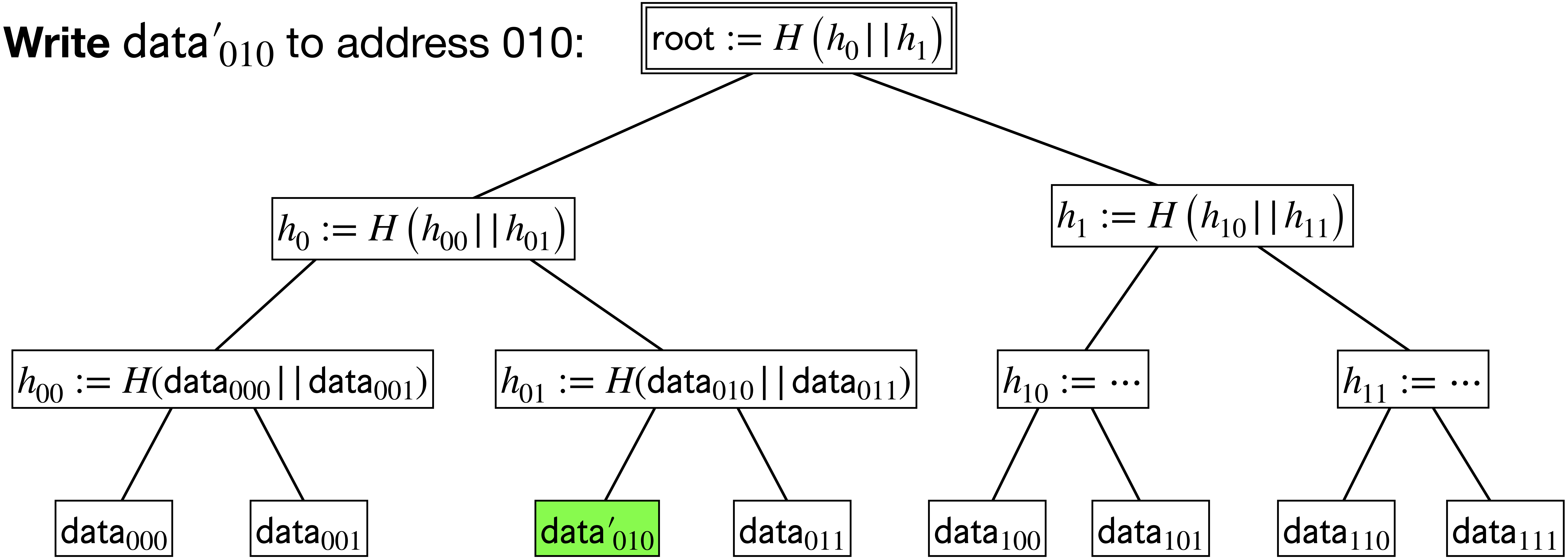


If all hashes to root are consistent, **return** data_{010}. Otherwise, **abort**.

Merkle Trees

Write data$'_{010}$ to address 010:



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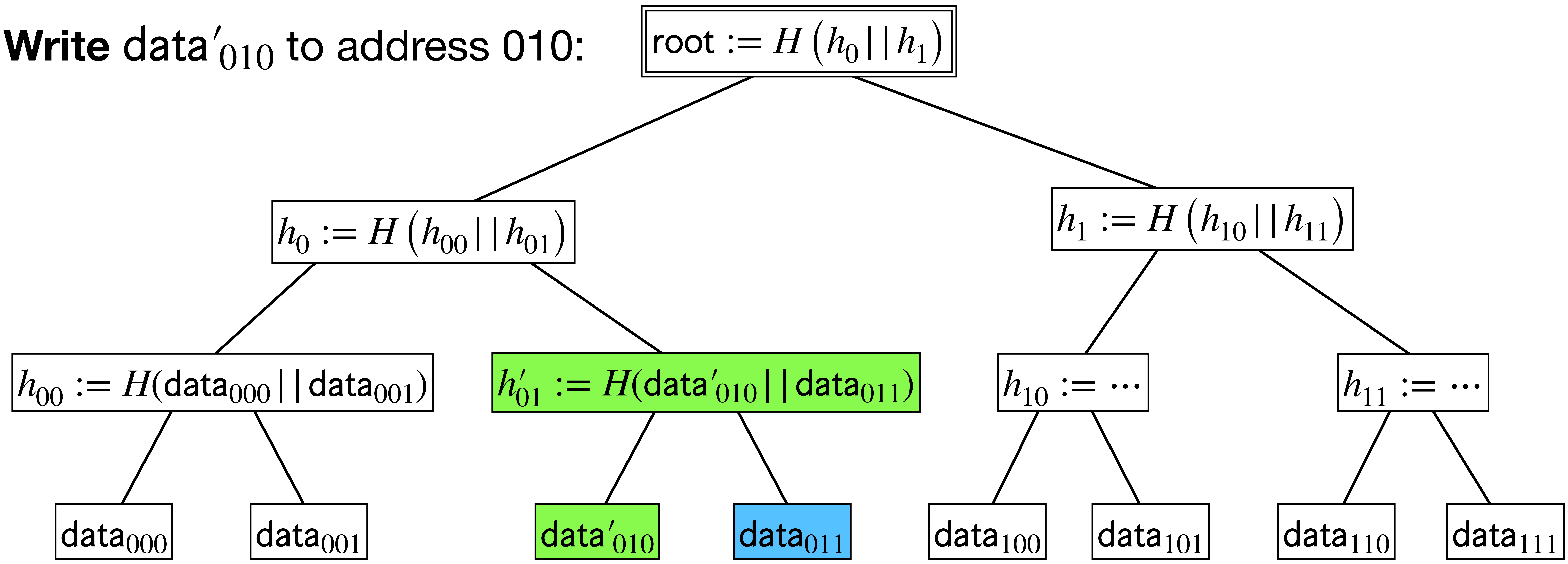

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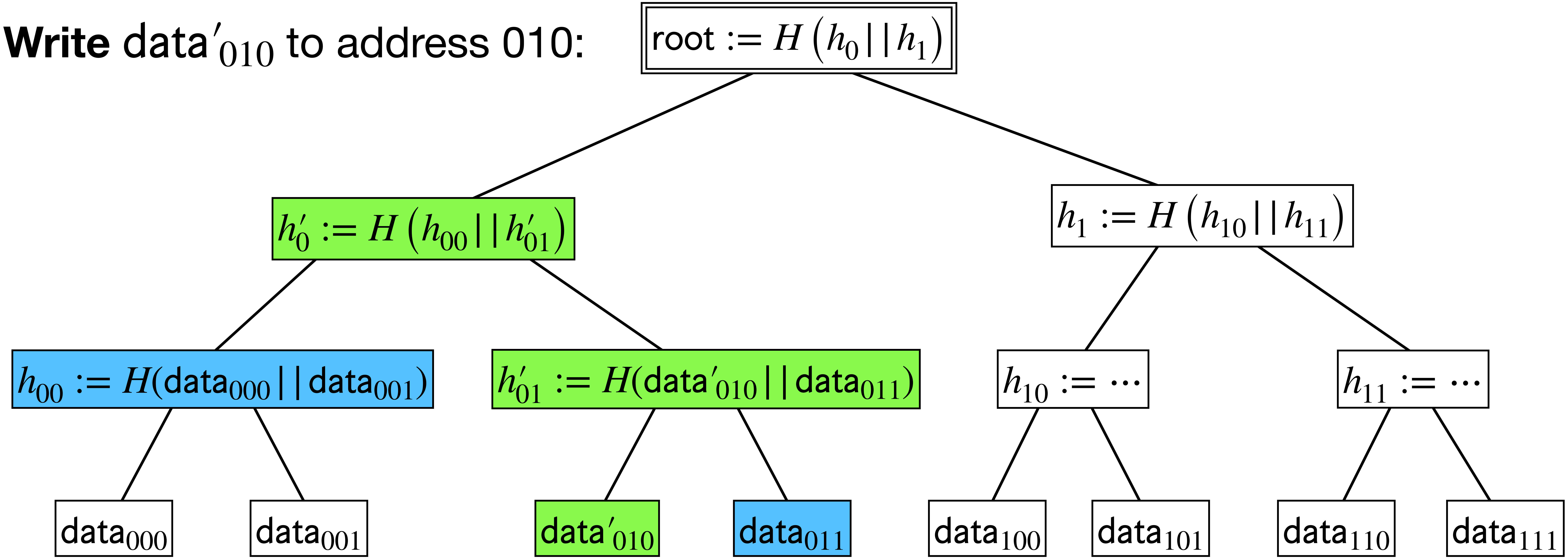

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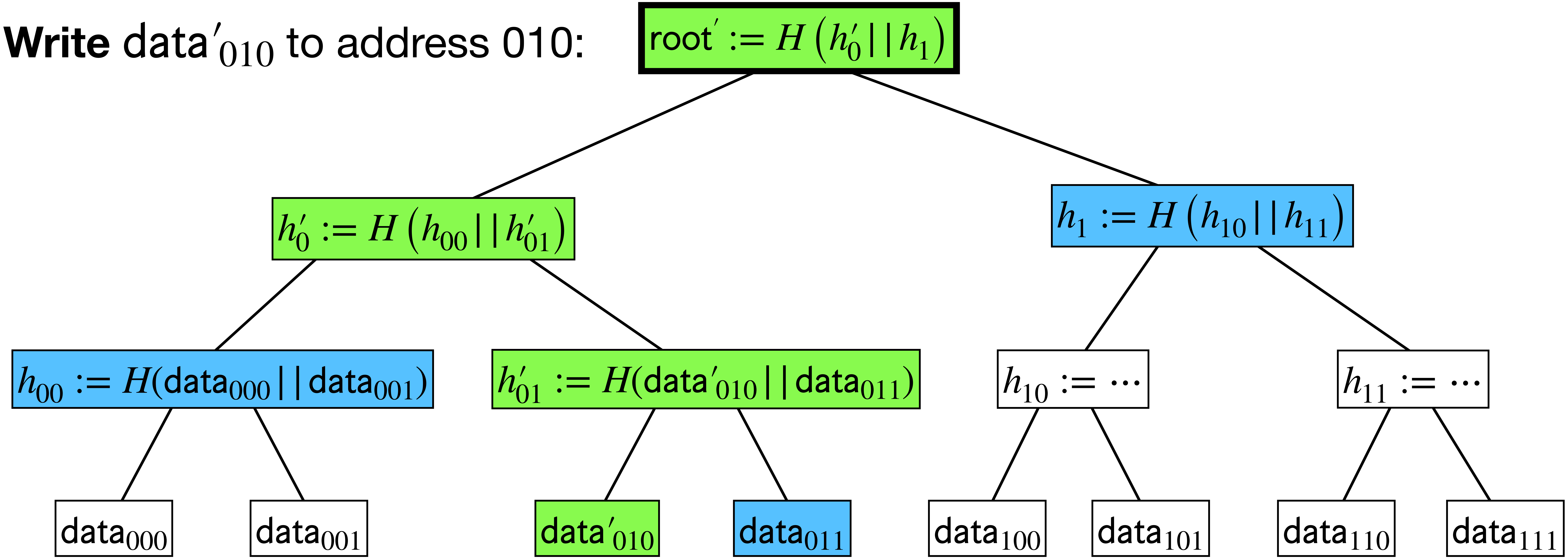

Merkle Trees

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  - **Local Space**: Hash root and key (can both be made small).

Merkle Trees

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Merkle Trees

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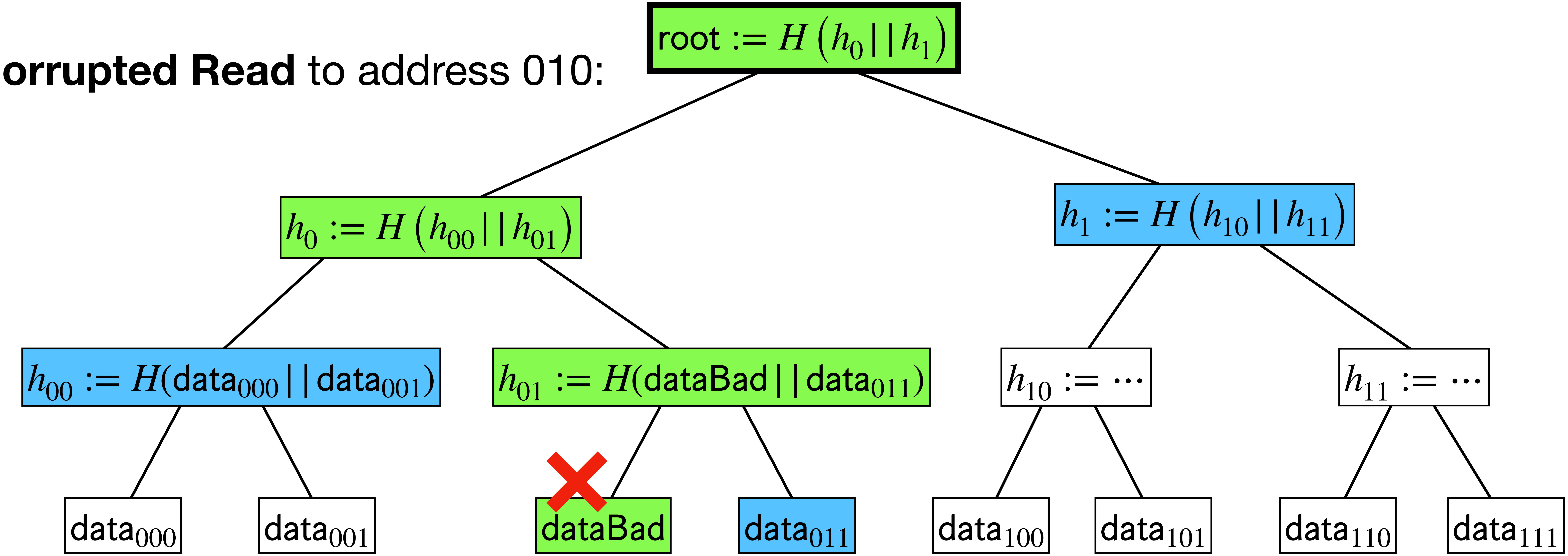
  - Suppose adversary cheats (undetectably forces wrong output on some read).
  - Consider first, top-most entry that adversary gives **wrong** hash value.
    - Can’t be the root, because we store the root locally.
  - This will be a hash collision!

Merkle Trees

Corrupted Read to address 010:



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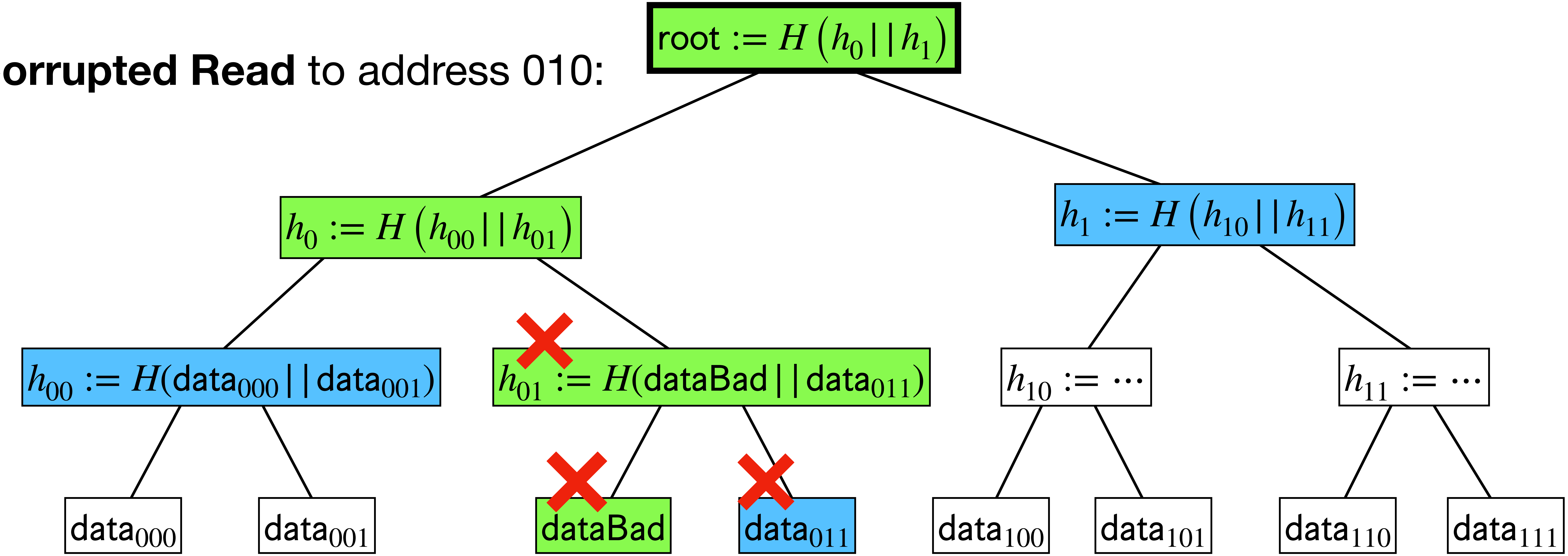

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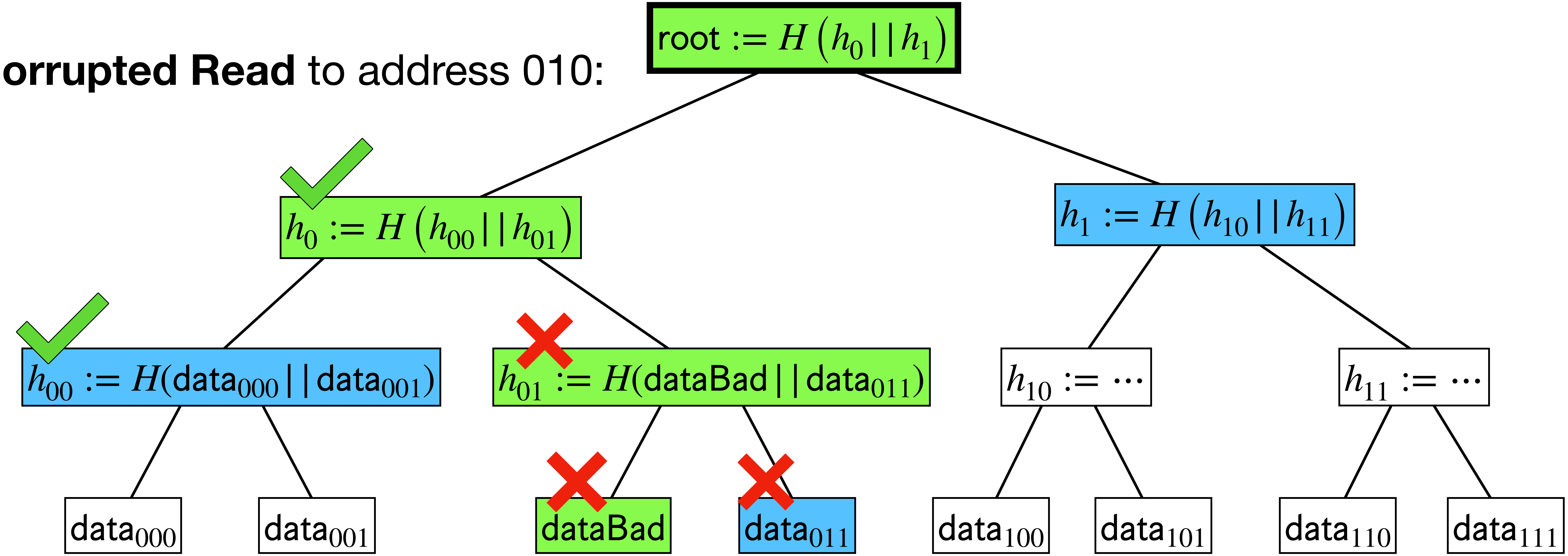

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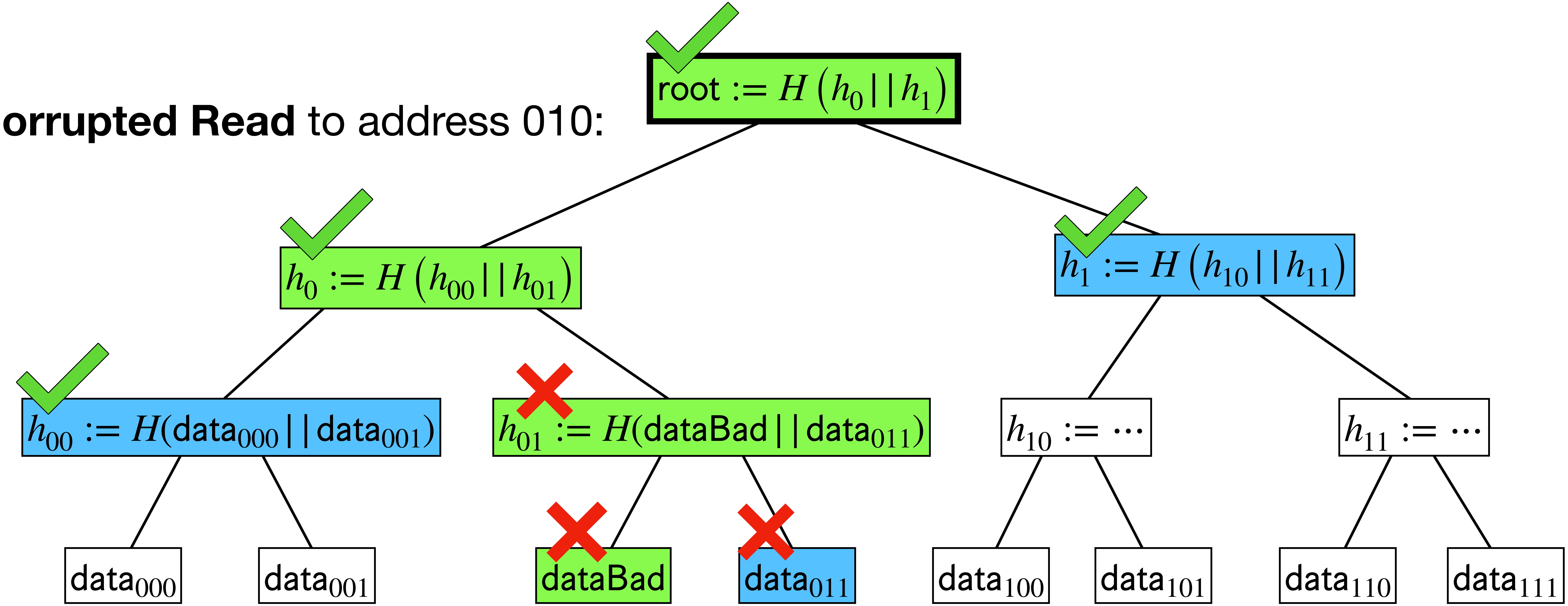

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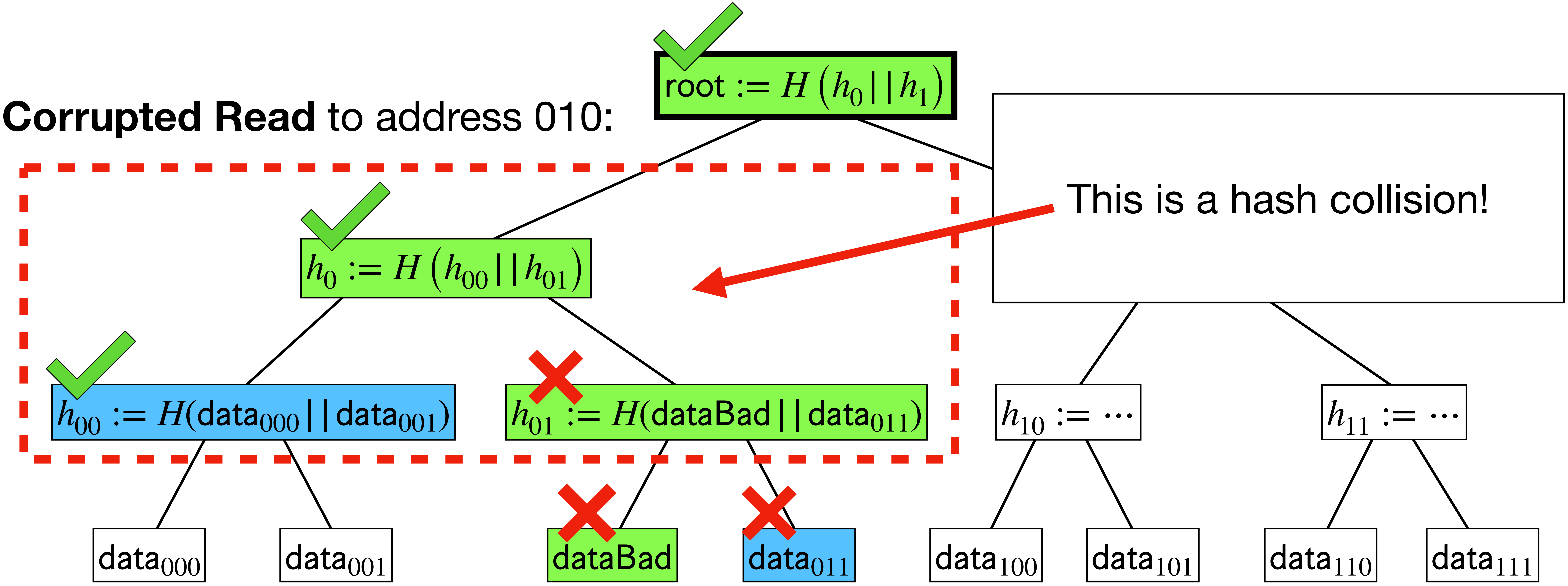

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1. Succinct Argument System for NP (Merkle trees + PCP theorem).

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3. Blockchains (e.g., bitcoin)!

Solving Privacy: Oblivious RAM

Oblivious RAM (Solving Privacy Issue)

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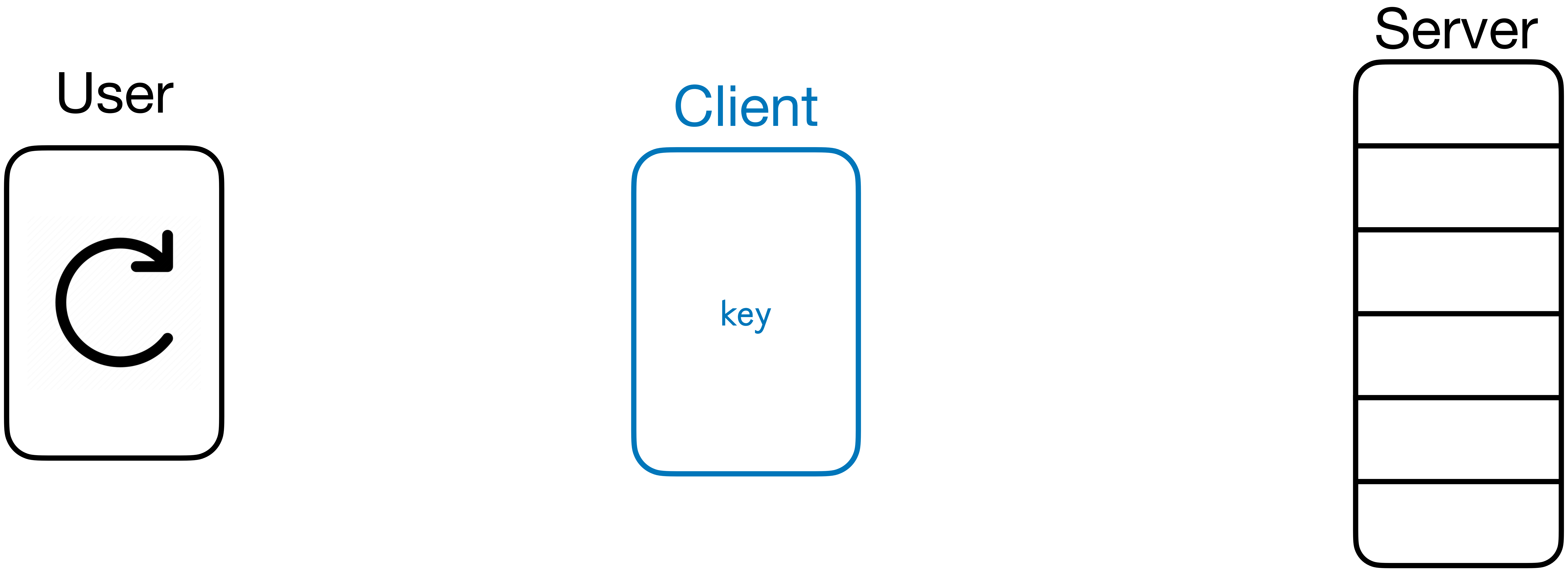
Wait, does encryption solve the privacy issue?

Encryption as ORAM?



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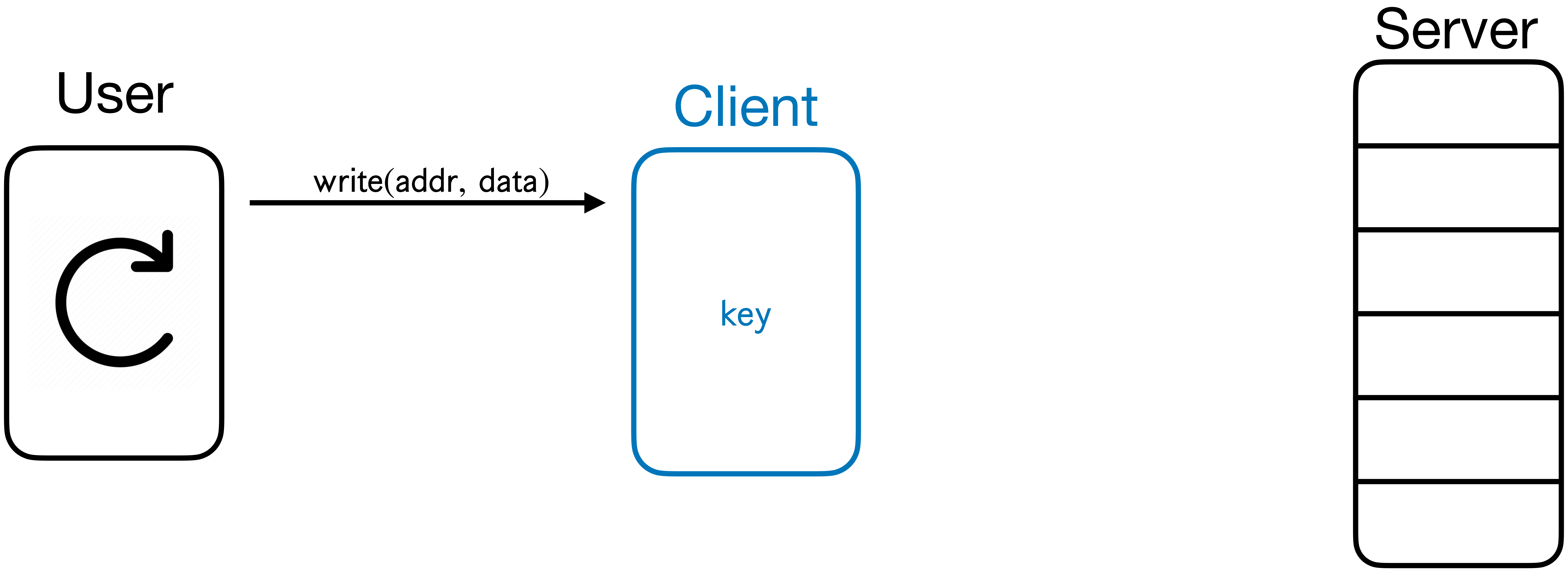

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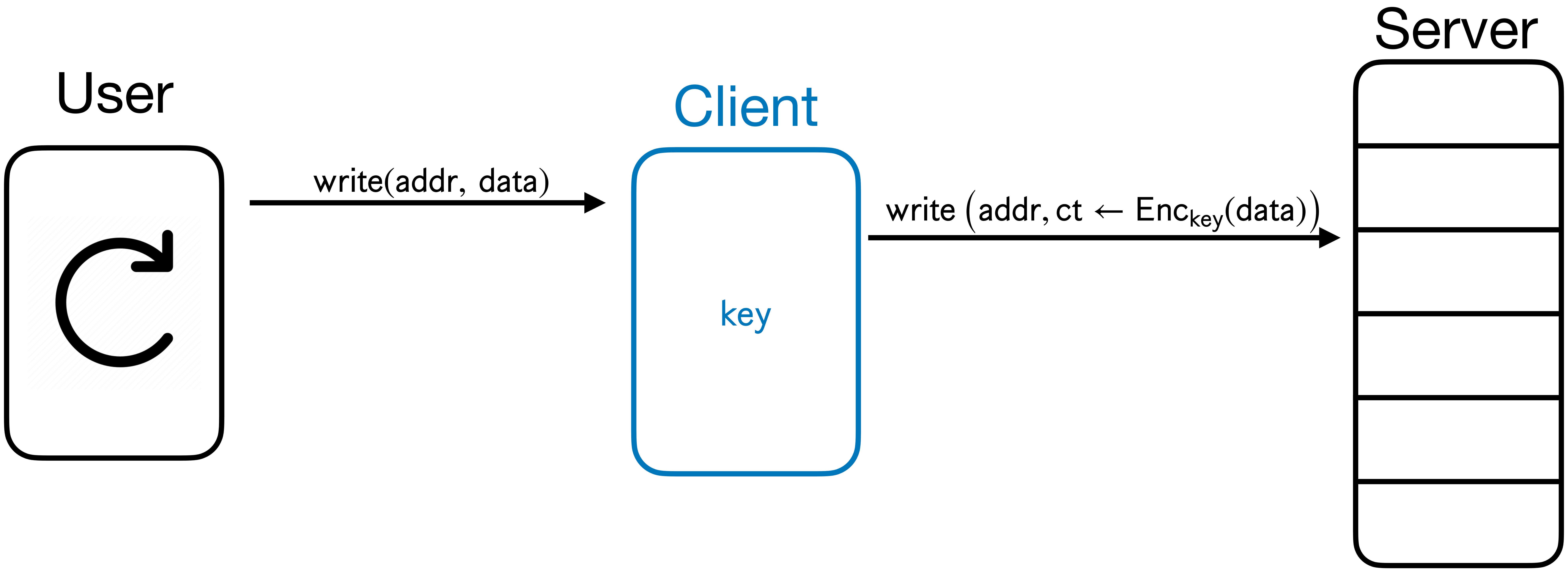

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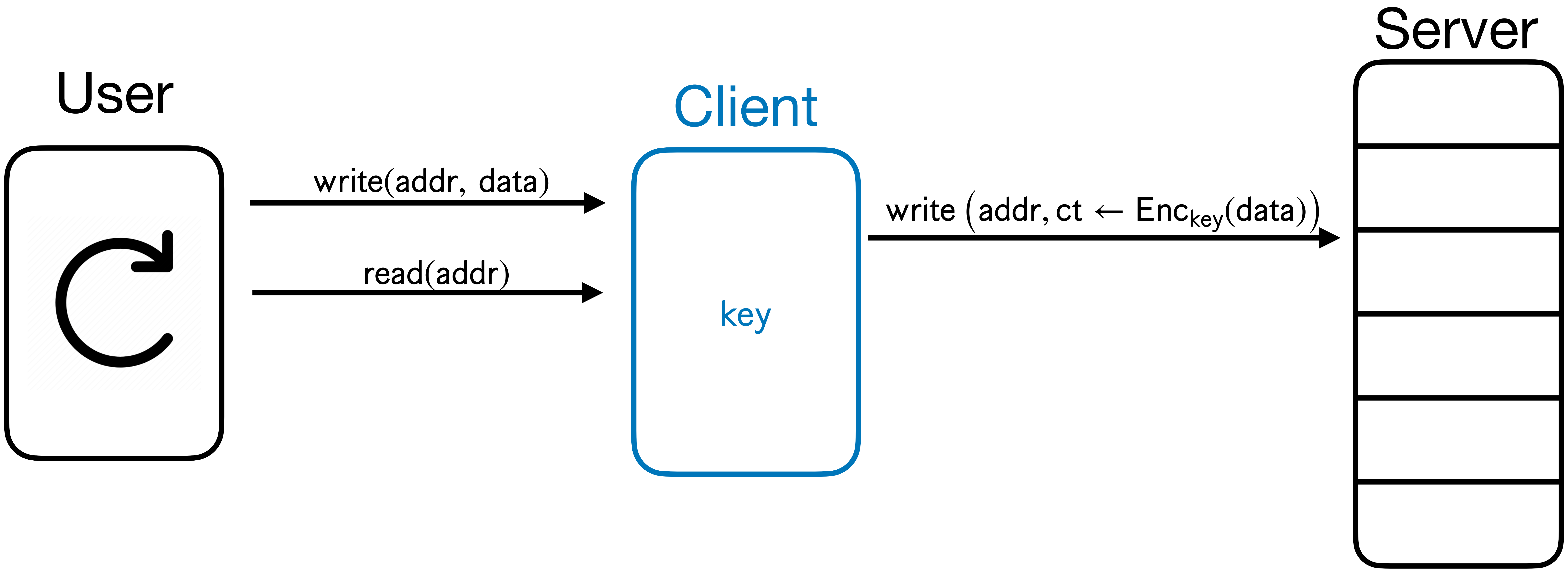

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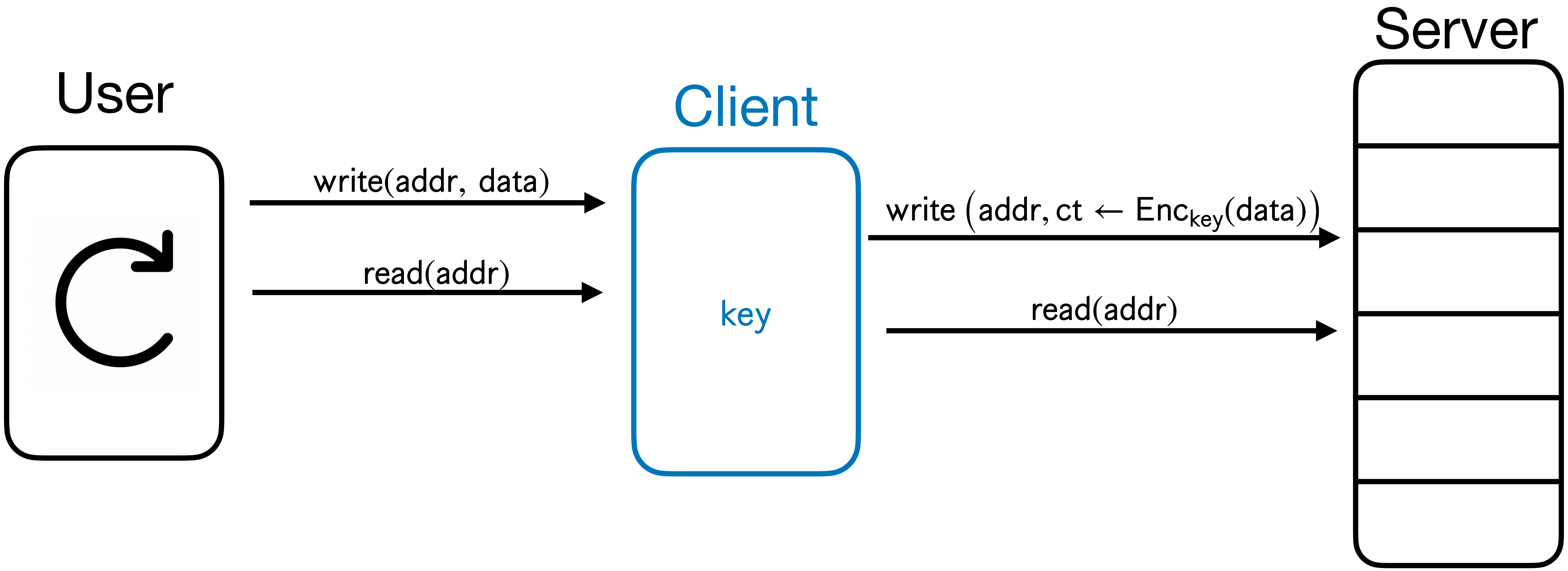

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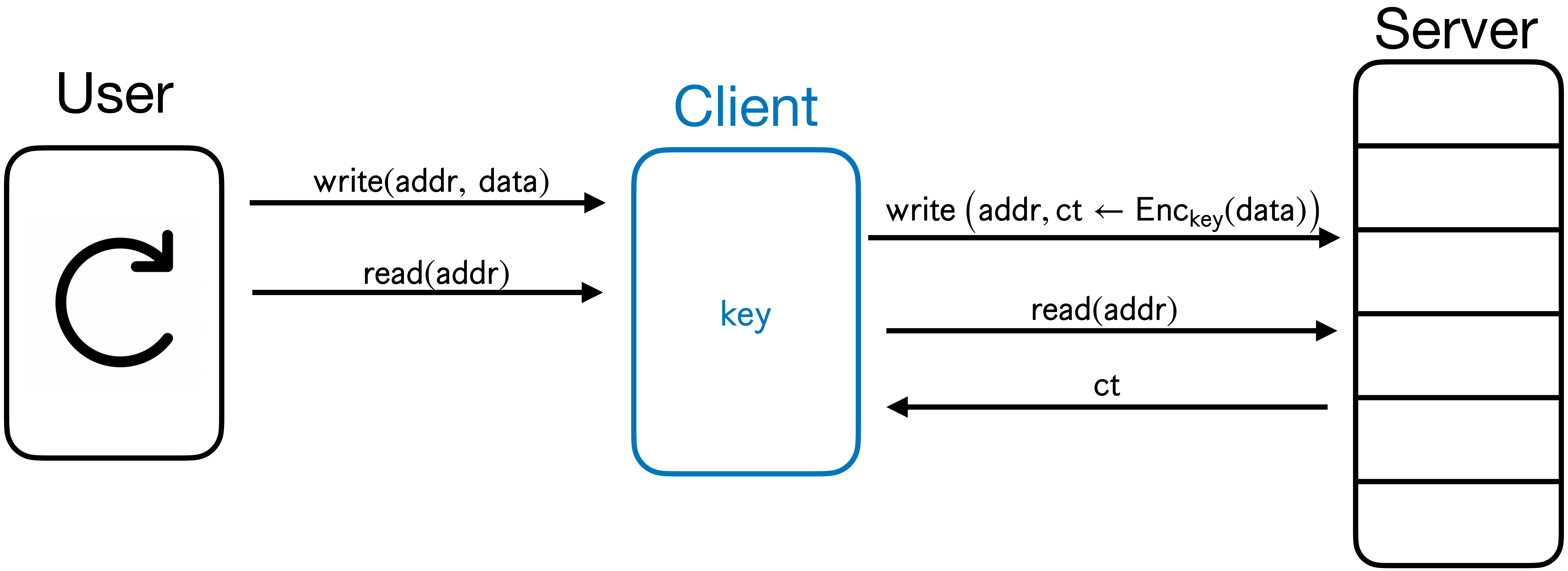

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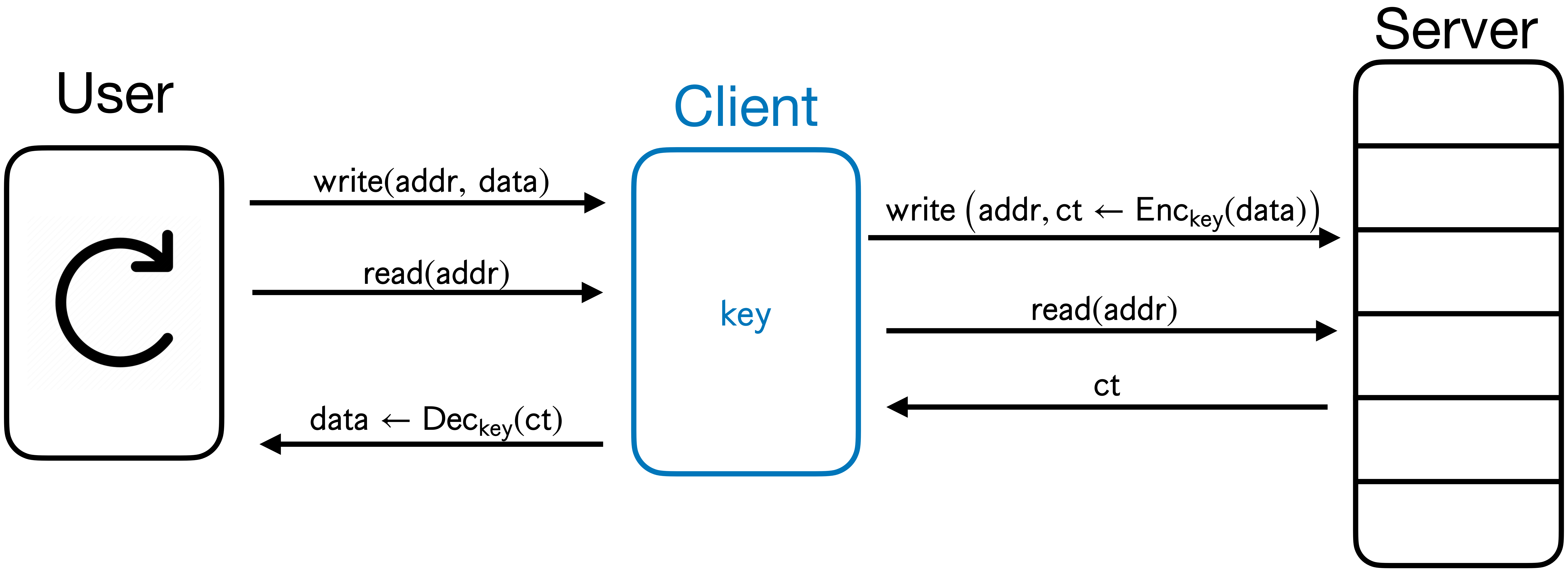

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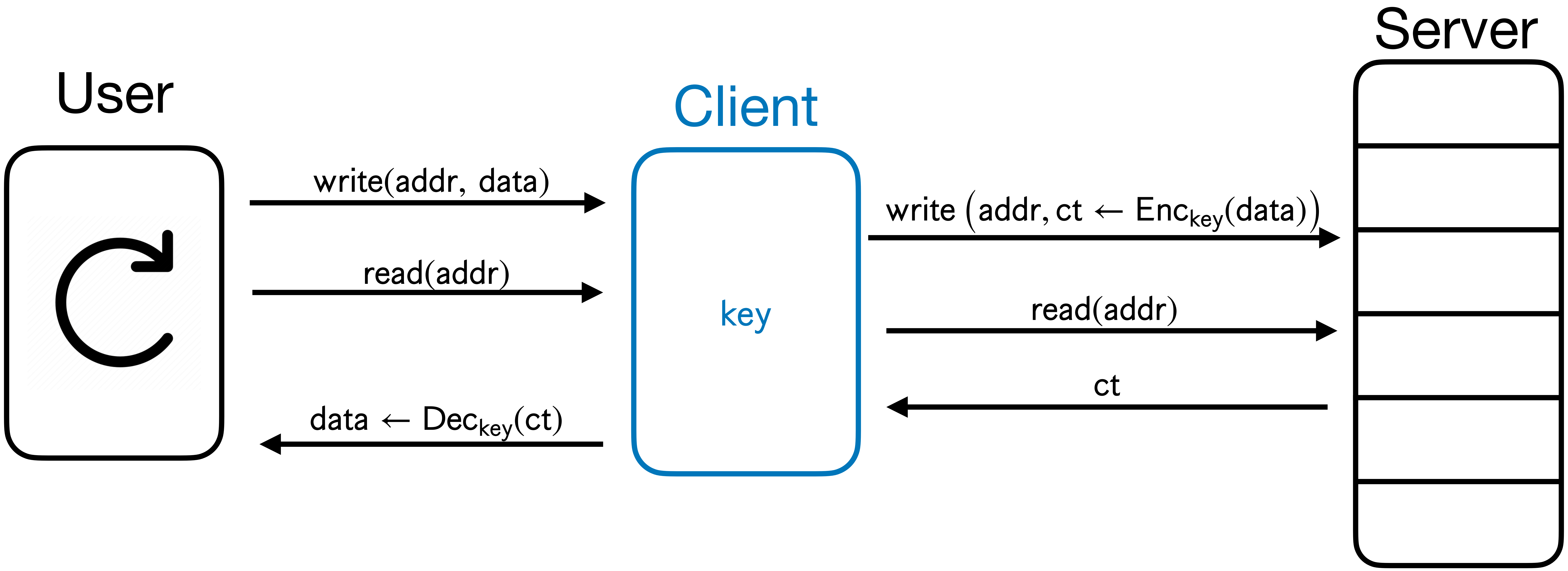

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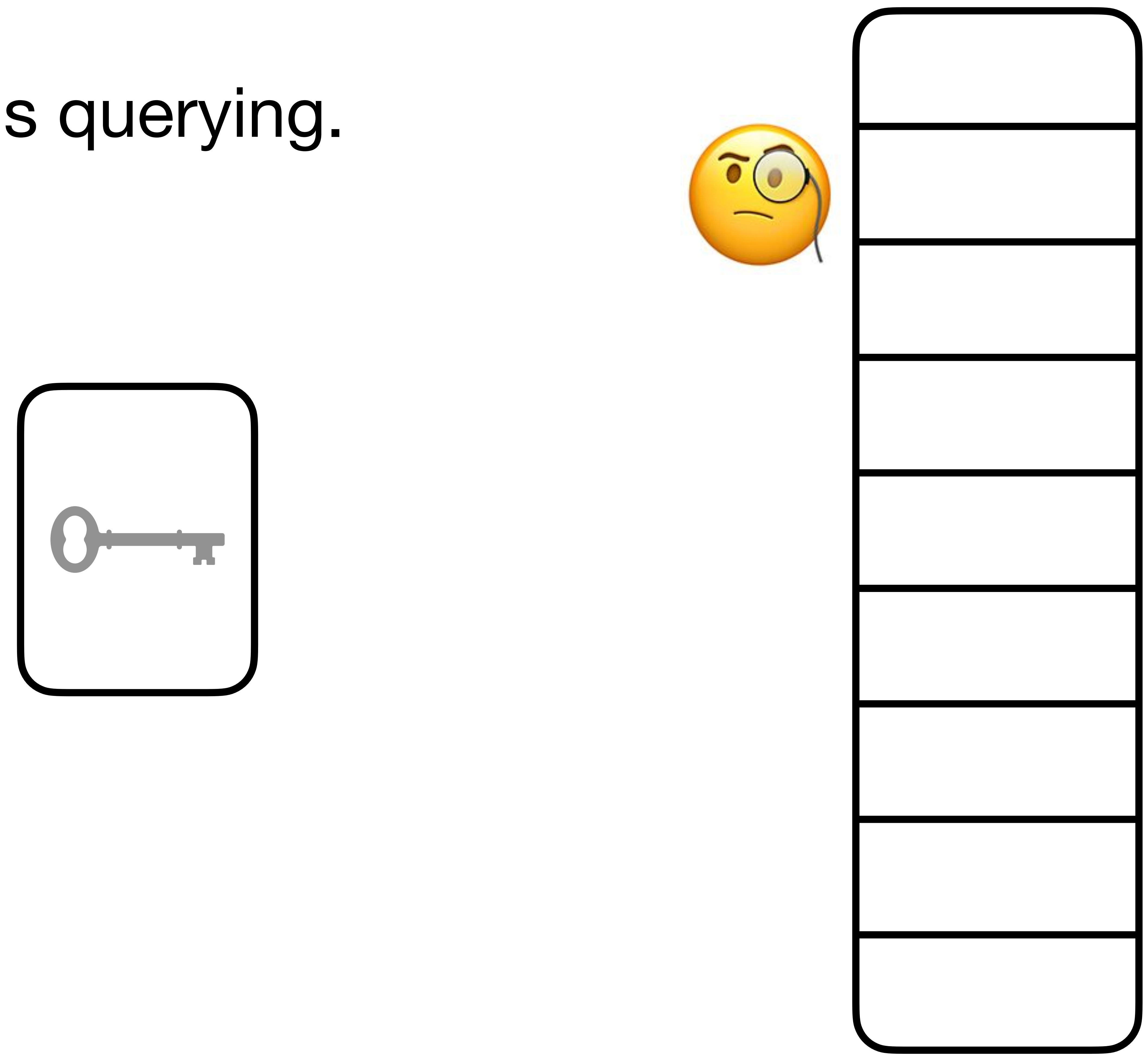

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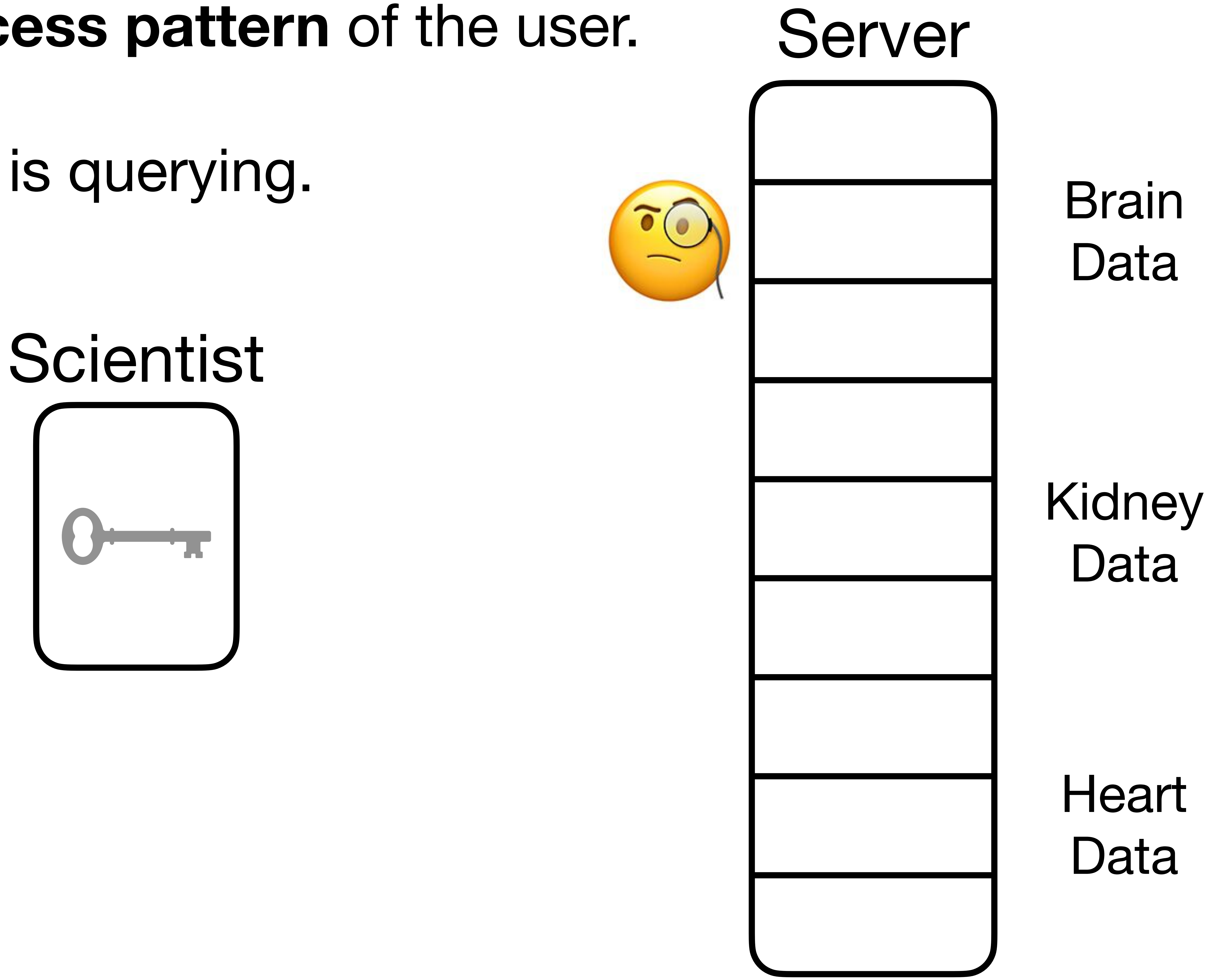

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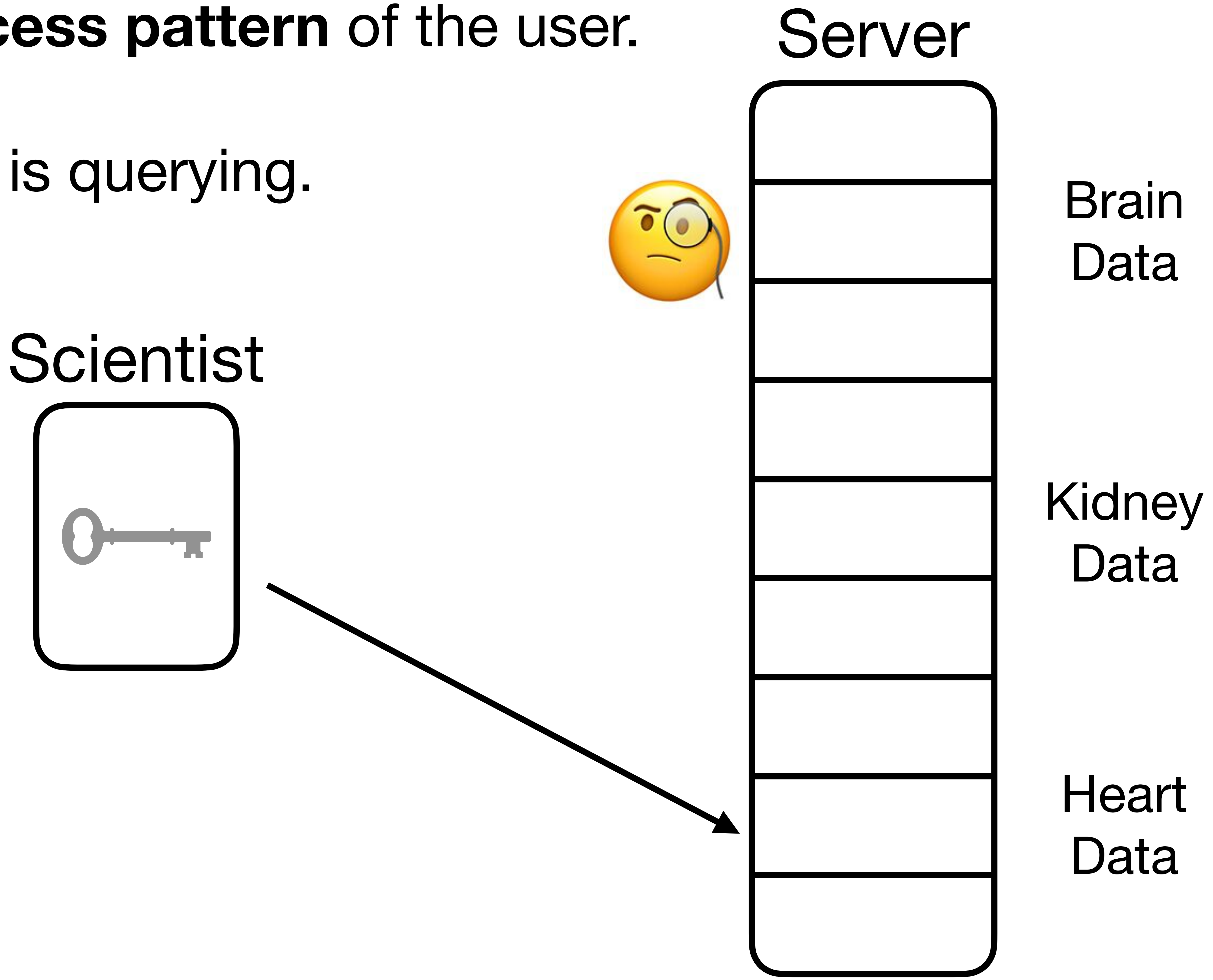

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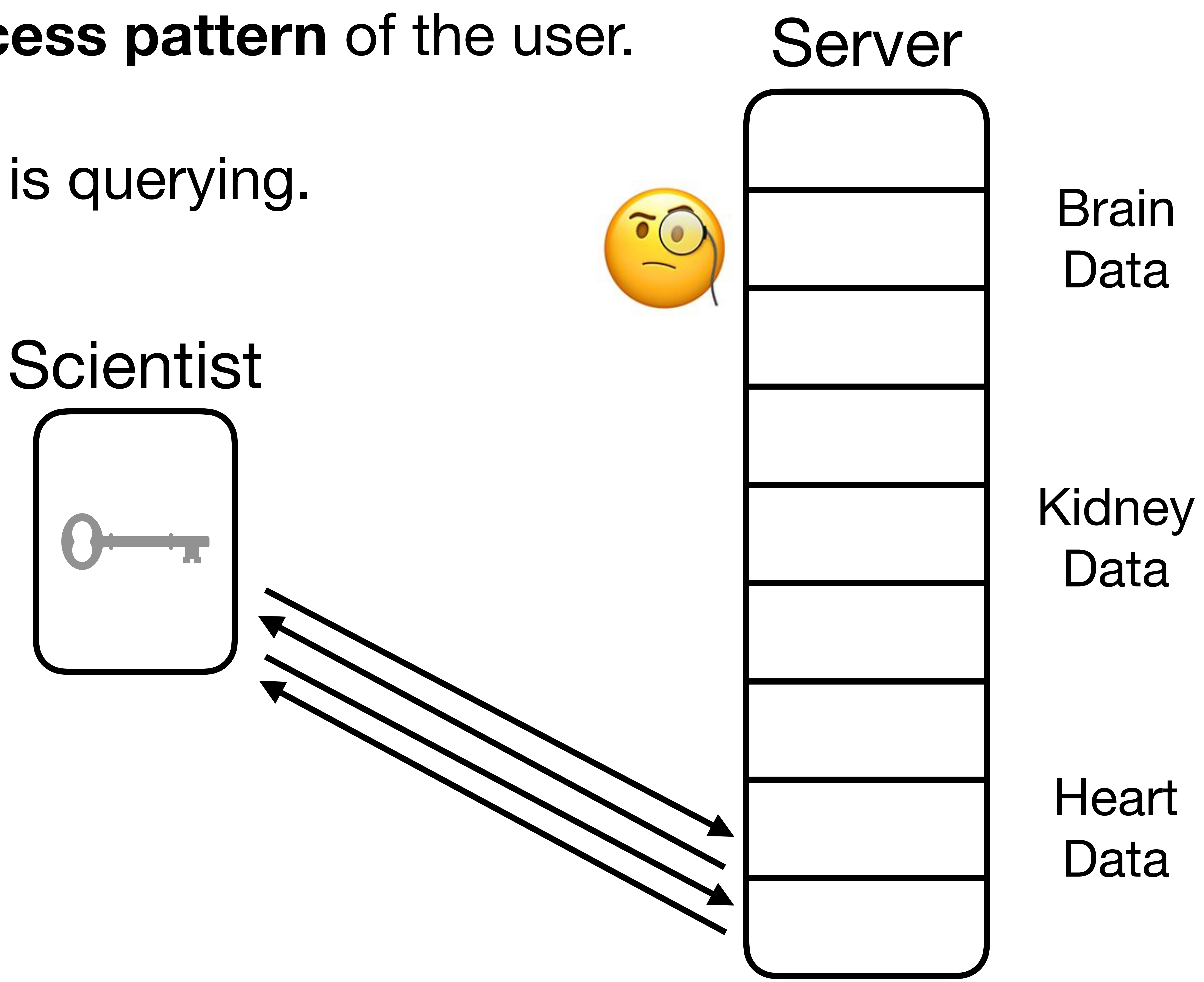

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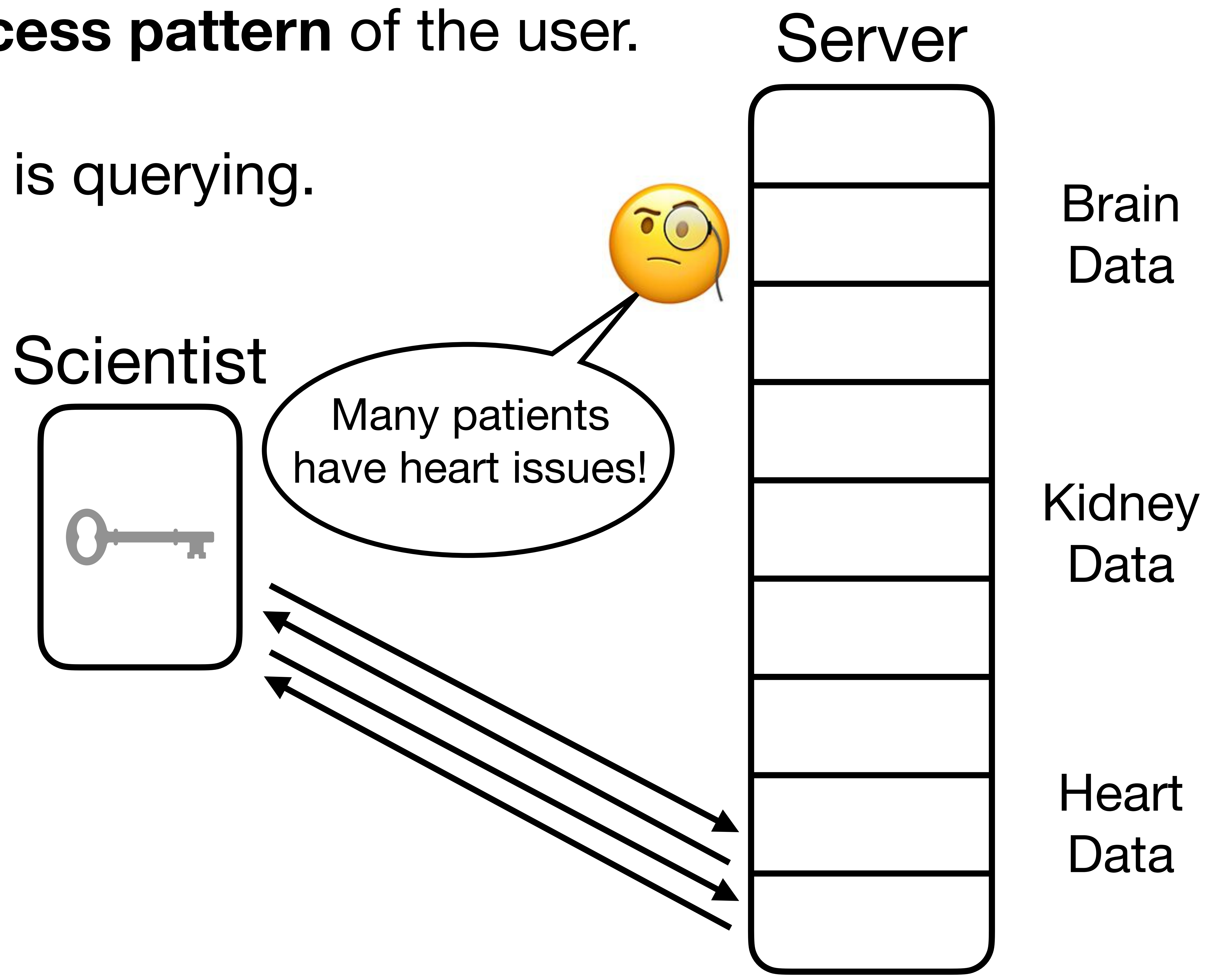

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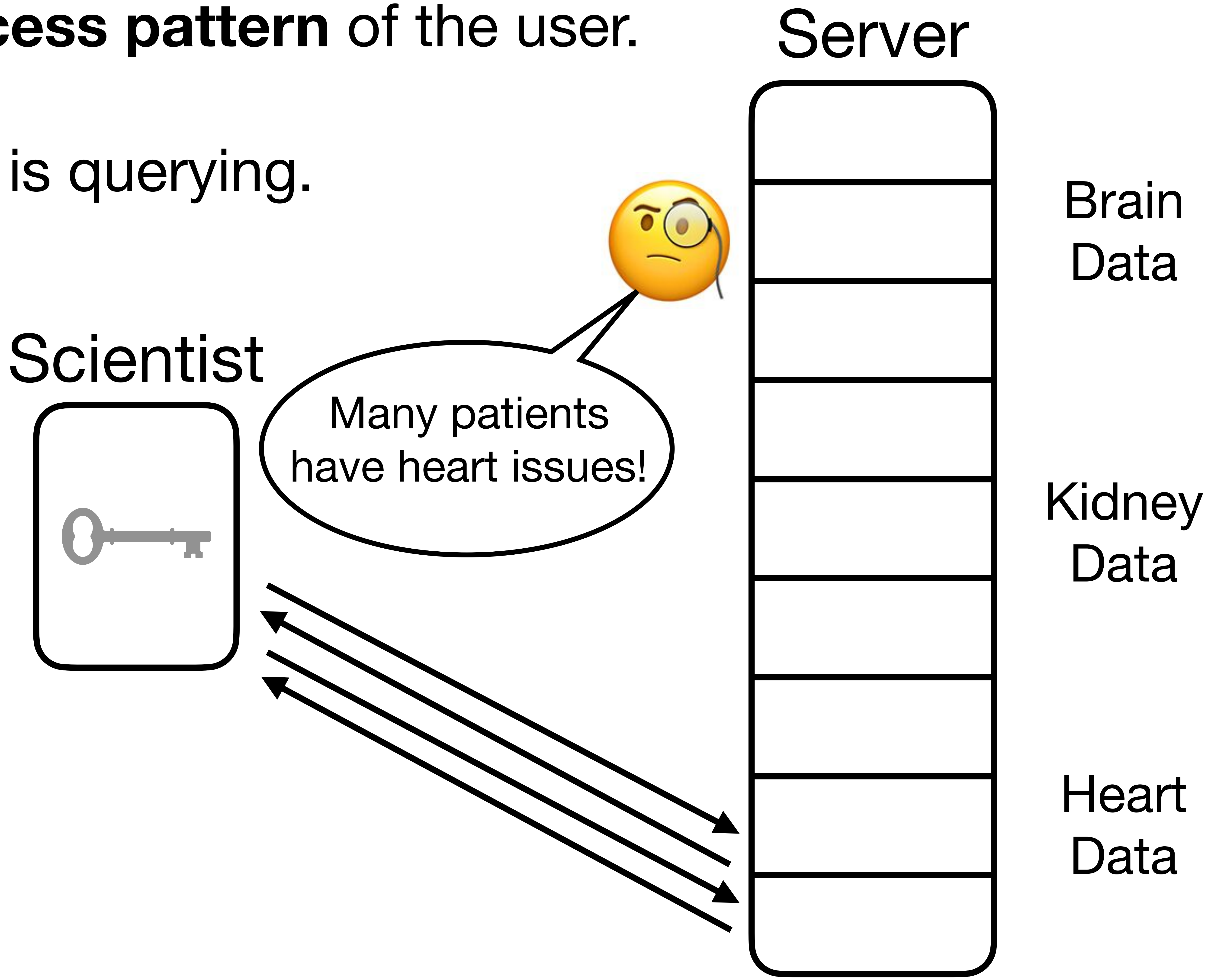
- This matters!
- **Real world example:** Adversary looking at accesses to encrypted email repository can recover as much as 80% of search queries [IKK '12].



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- Idea: “freshly” randomize address space each time.

Path ORAM

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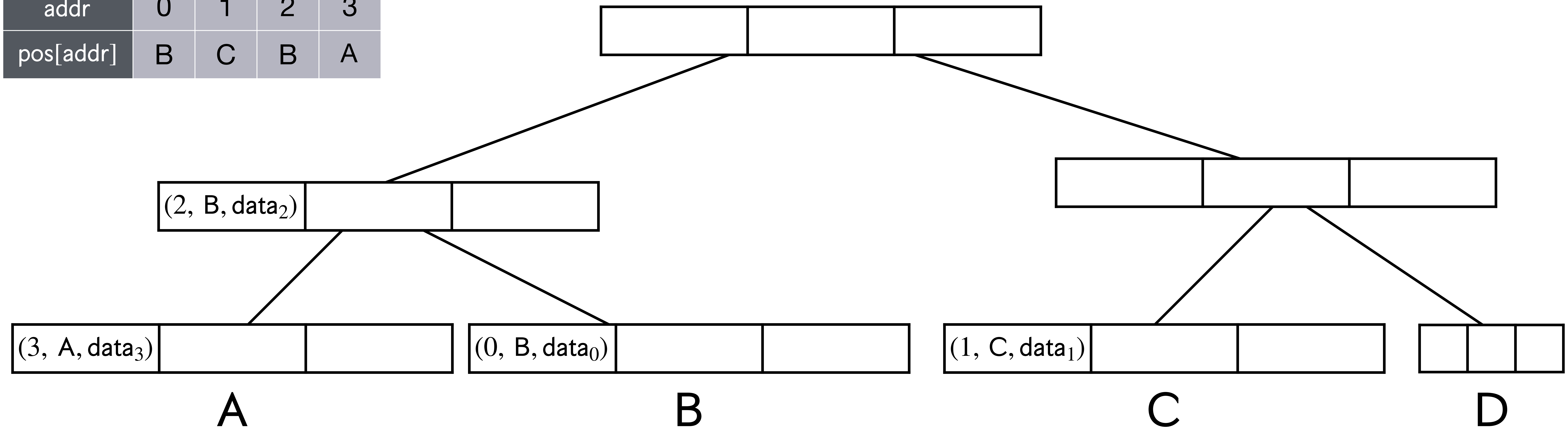
- Each data block consists of $(\mathsf{addr}, \mathsf{pos}[\mathsf{addr}], \mathsf{data})$.

Path ORAM

(Here, $N = 4$.)

addr	0	1	2	3
pos[addr]	B	C	B	A



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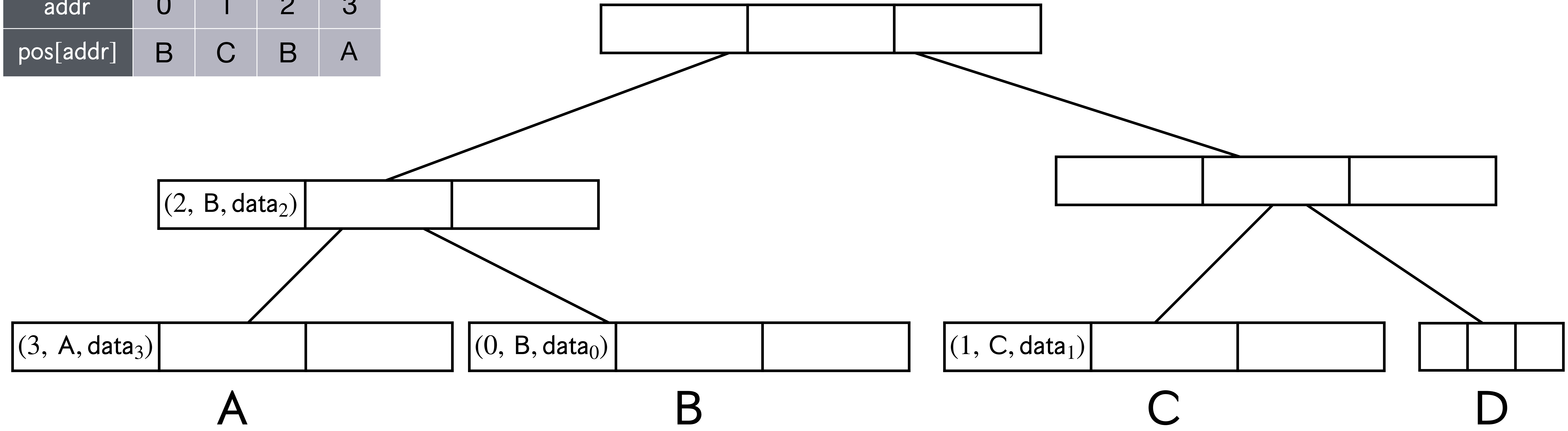

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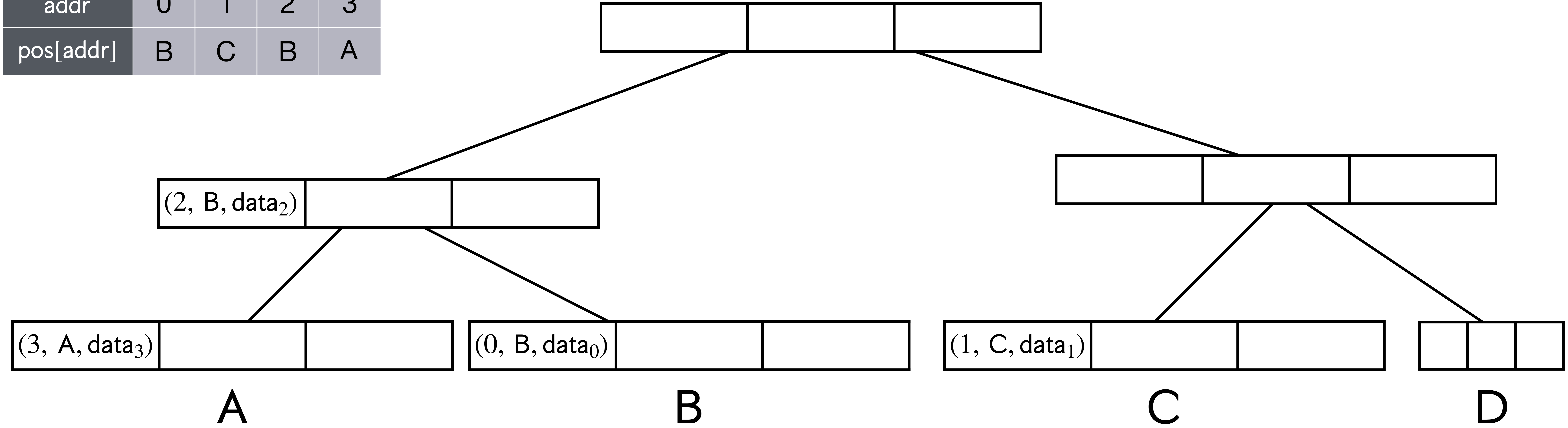

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1. **Look up** pos[addr] $\in \{A, B, C, D\}$ locally.

Path ORAM

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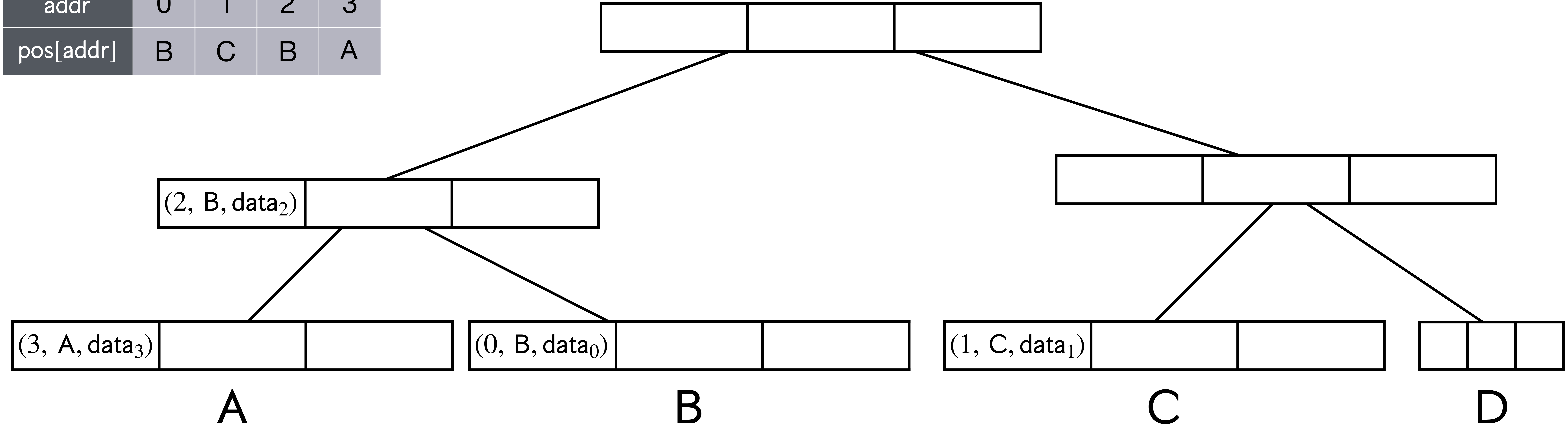

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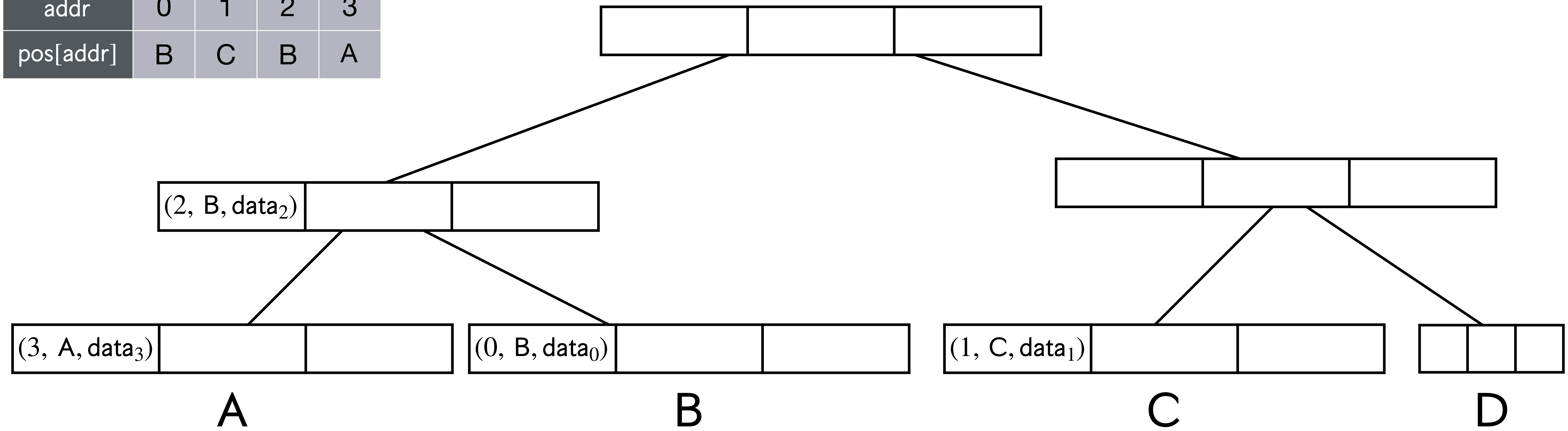

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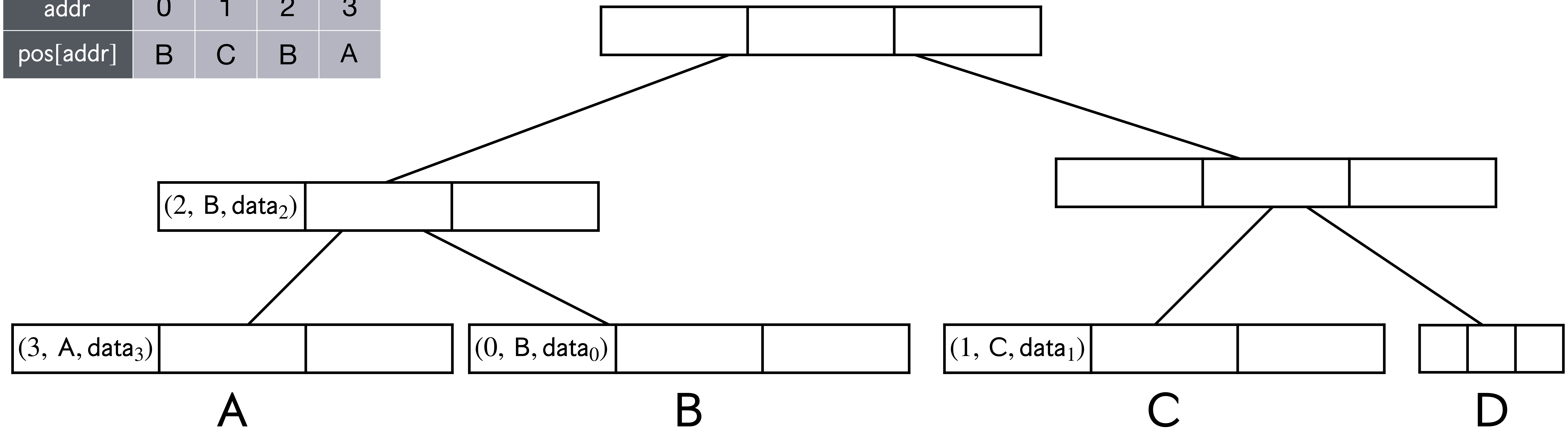

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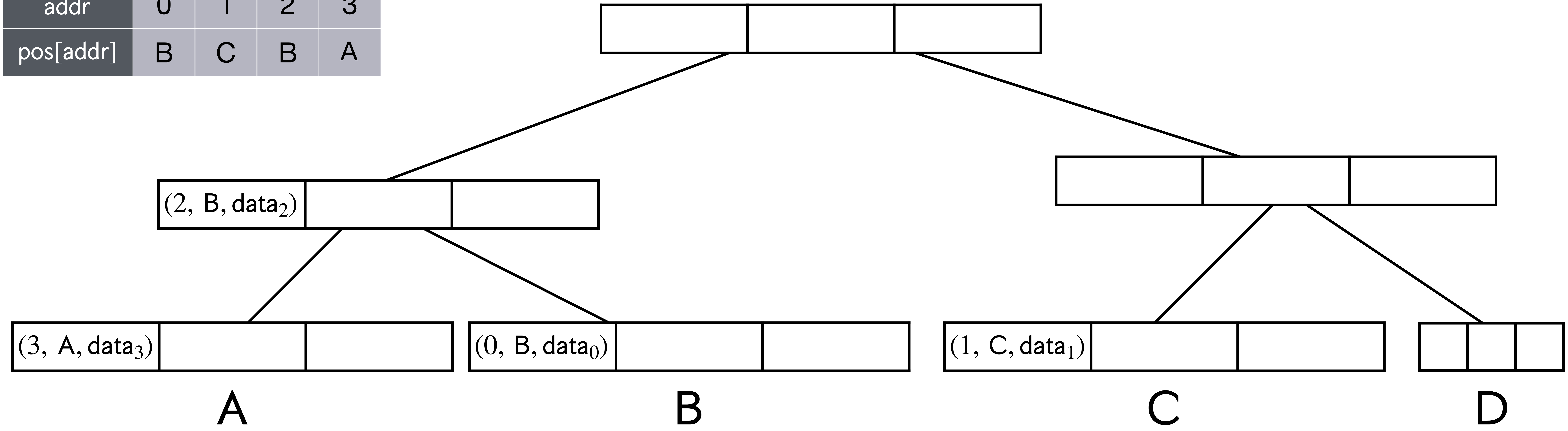

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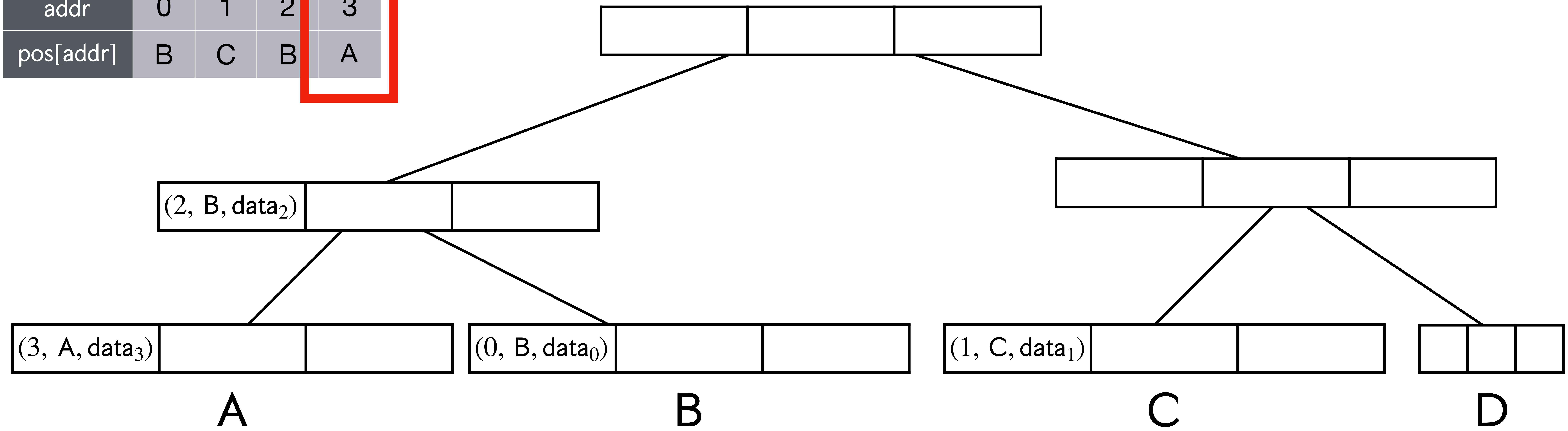

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Example: Read addr $= 3$

1. **Look up** pos$[3] = \mathsf{A}$ locally.

Path ORAM

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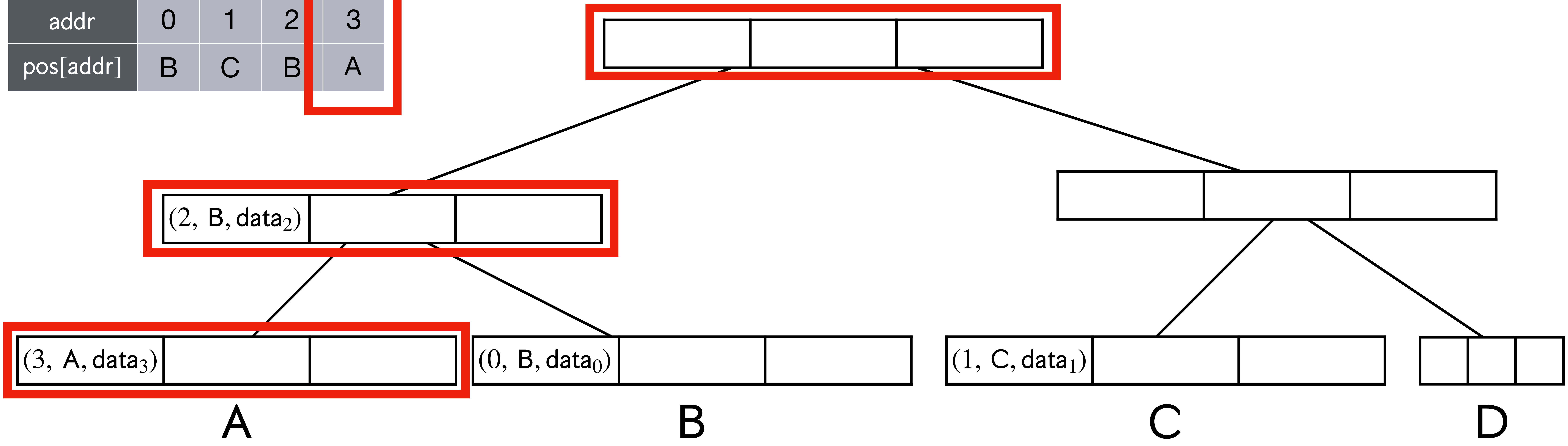
pos[addr]	B	C	B	A



On each query to addr $\in \{0, 1, 2, 3\}$:

1. **Look up** pos[addr] $\in$ {A, B, C, D} locally.
2. **Read** full path for leaf pos[addr].
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Example: Read addr $= 3$

1. **Look up** pos$[3]$ = A locally.
2. **Read** full path for leaf A.

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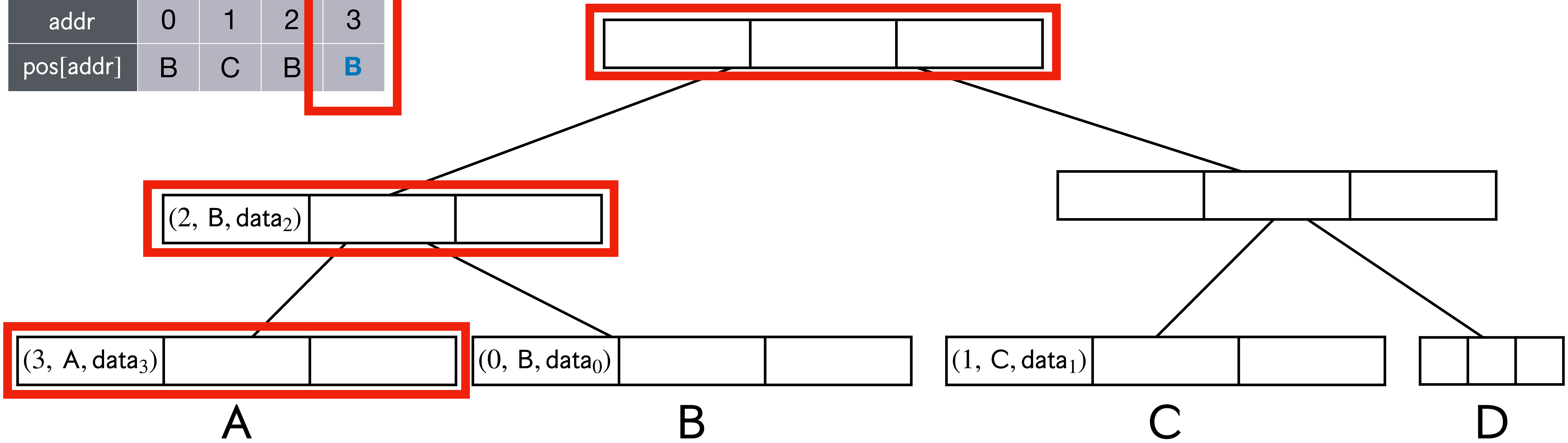
addr	0	1	2	3
pos[addr]	B	C	B	B



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1. **Look up** pos$[3] =$ A locally.
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3. Set pos$[3] \leftarrow \{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}\}$ randomly, say B.

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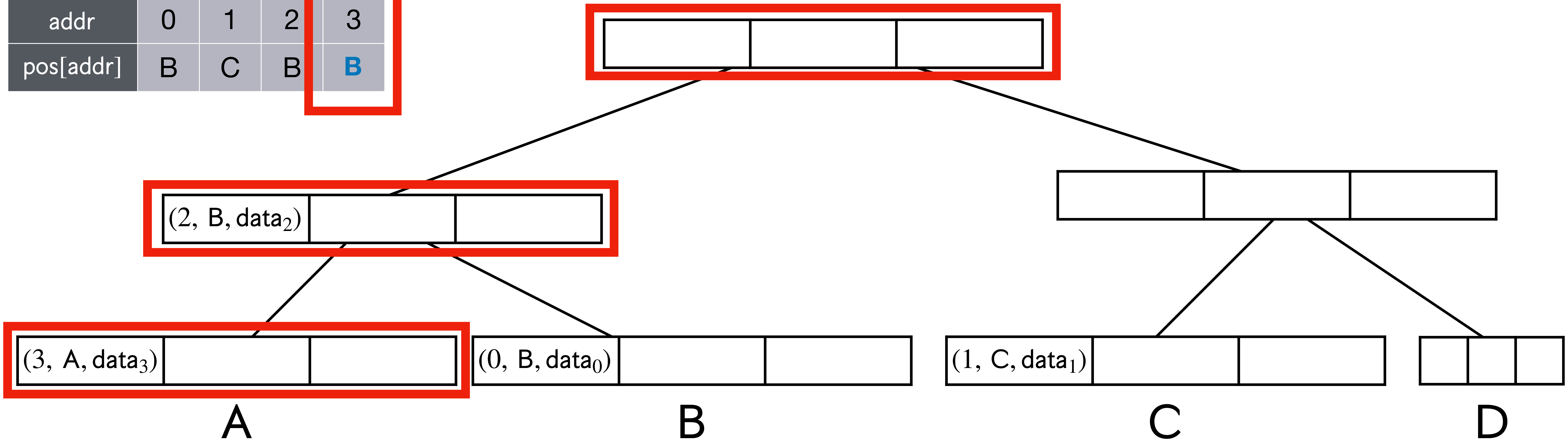
pos[addr]	B	C	B	**B**



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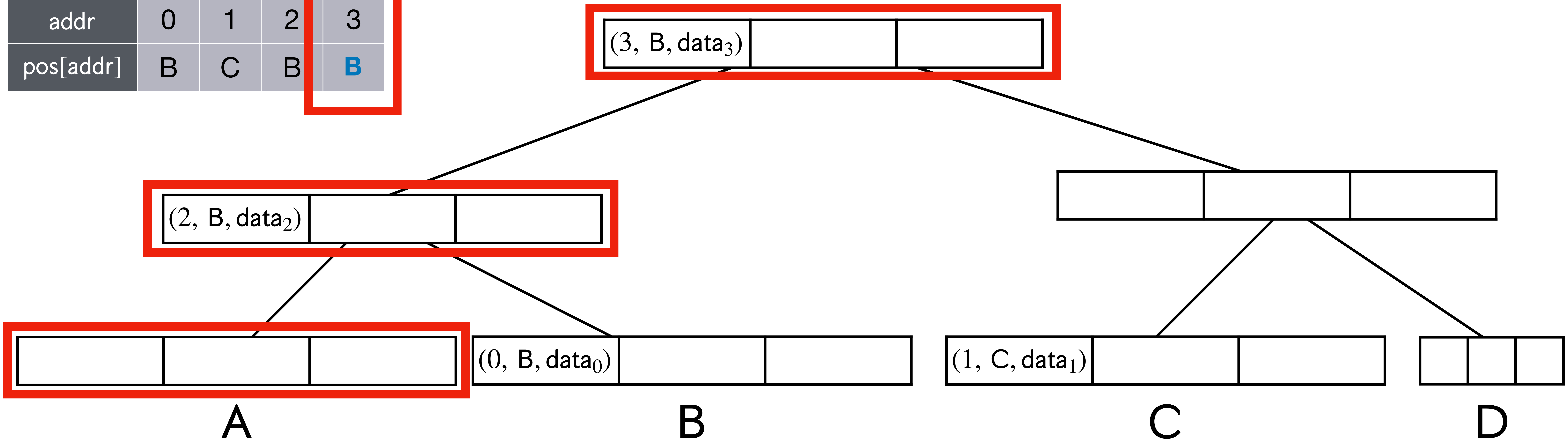
Example: Read addr $= 3$

1. **Look up** pos$[3] =$ A locally.
2. **Read** full path for leaf A.
3. Set pos$[3] \leftarrow \{A, B, C, D\}$ randomly, say B.
4. **Push new** (and other) blocks down A path.

Path ORAM

(Here, $N = 4$.)

addr	0	1	2	3
pos[addr]	B	C	B	B



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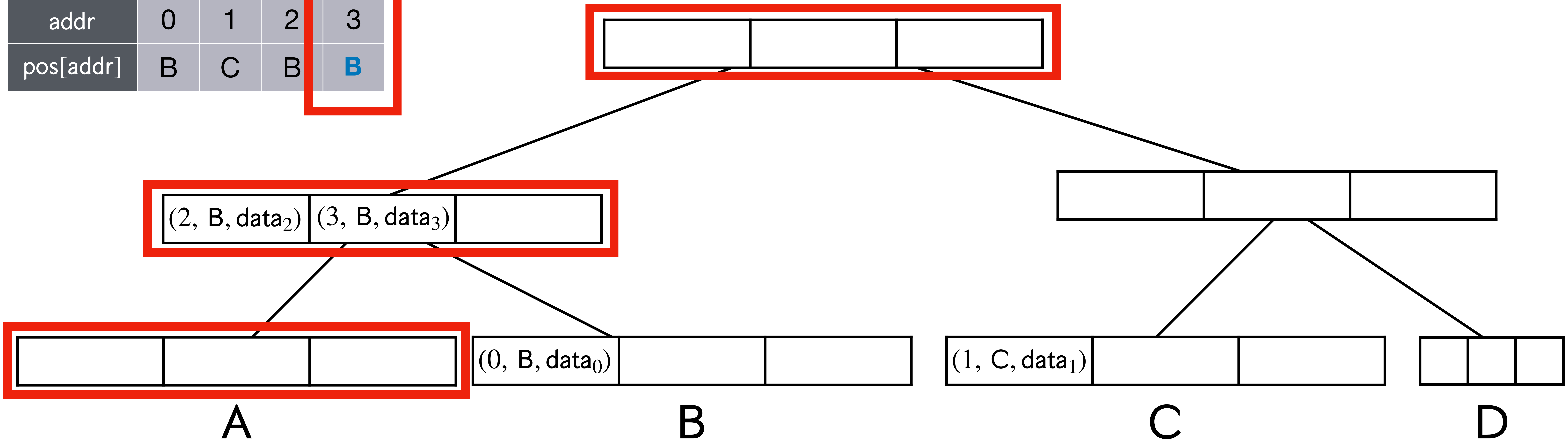

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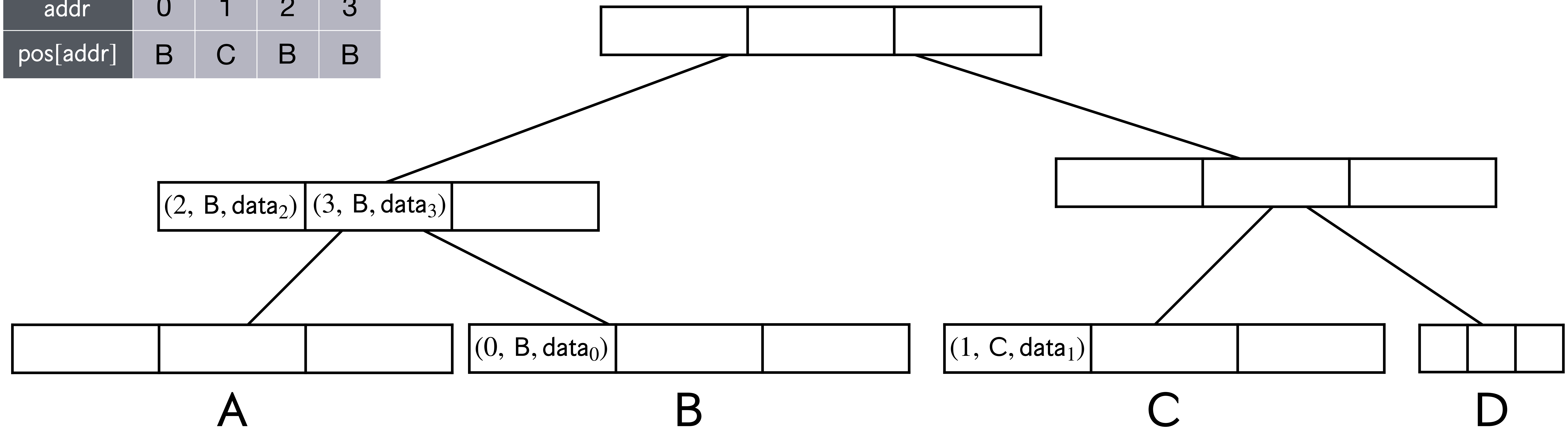

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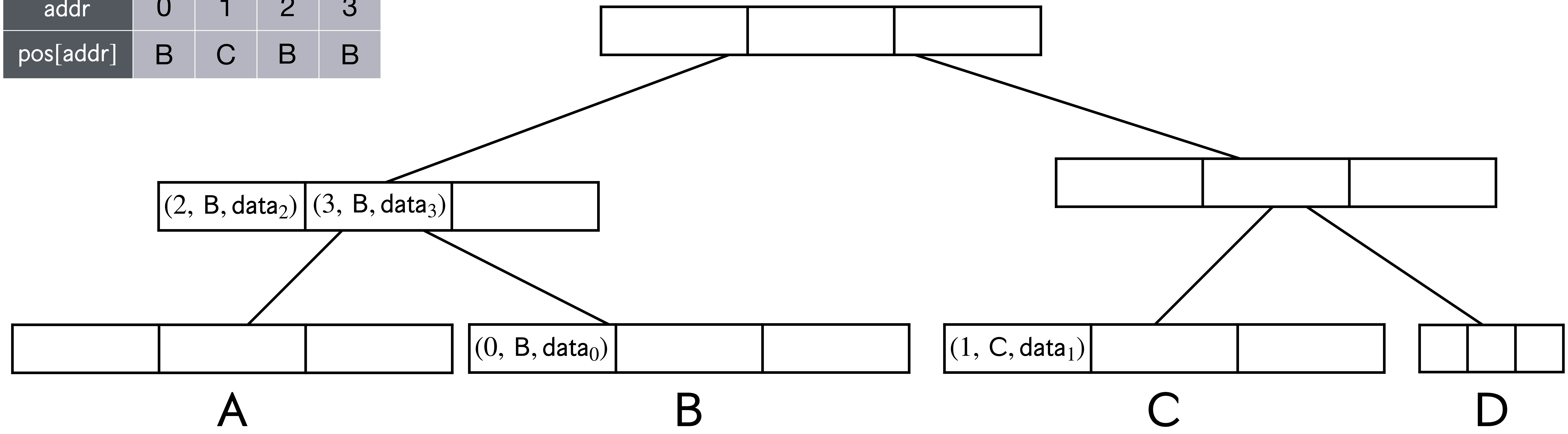

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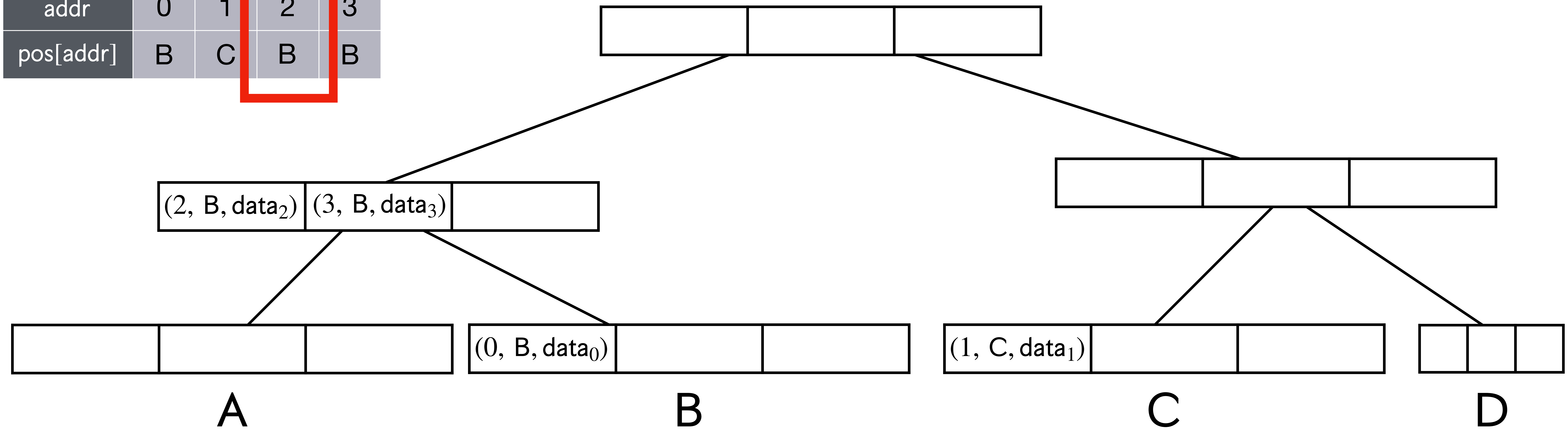

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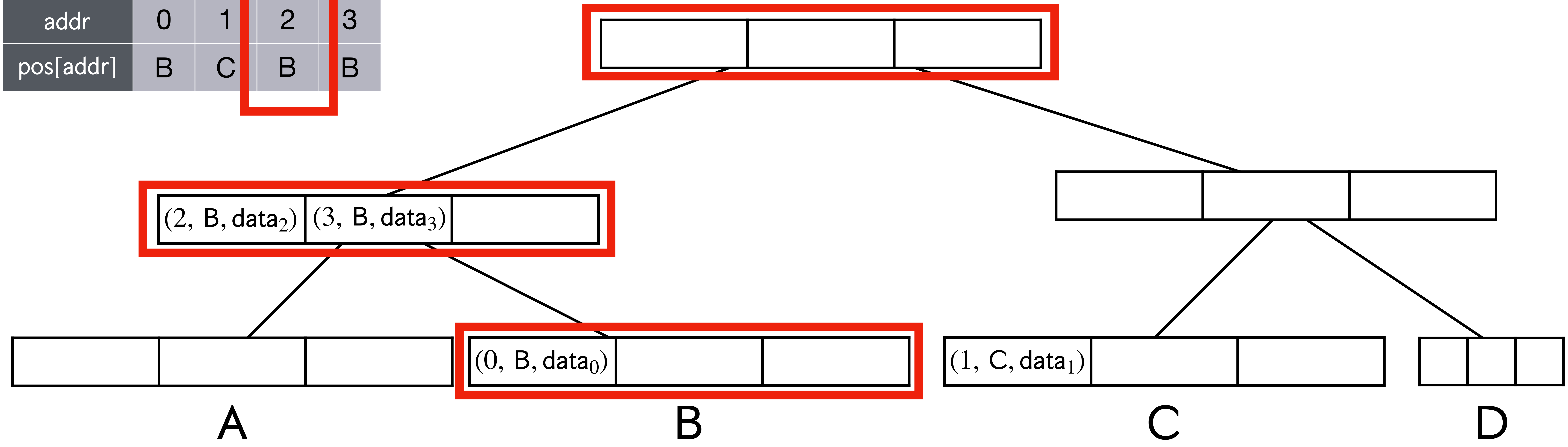

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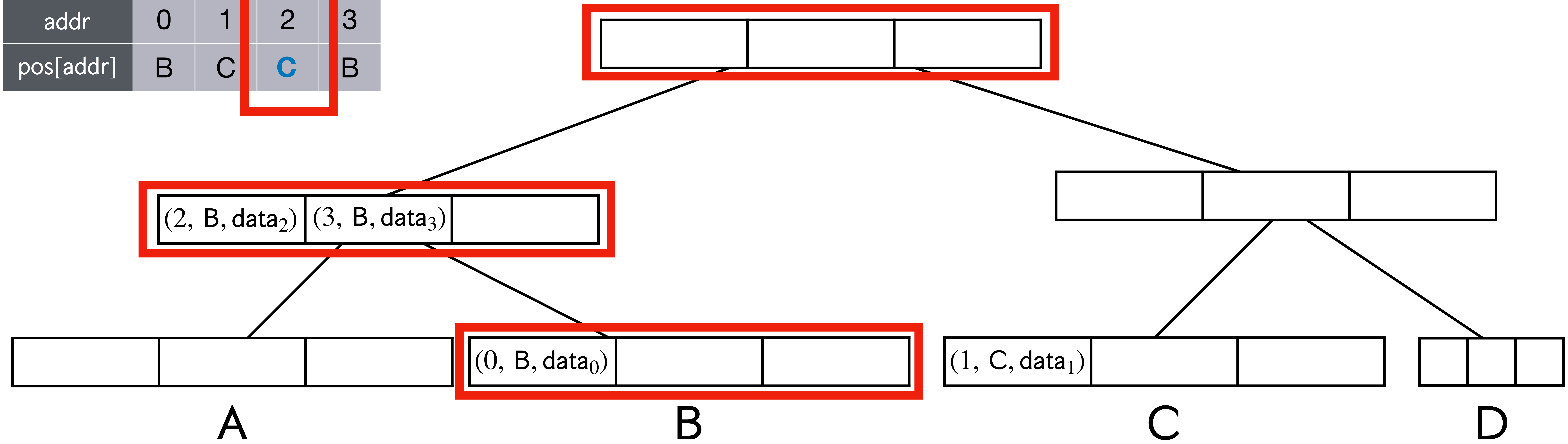

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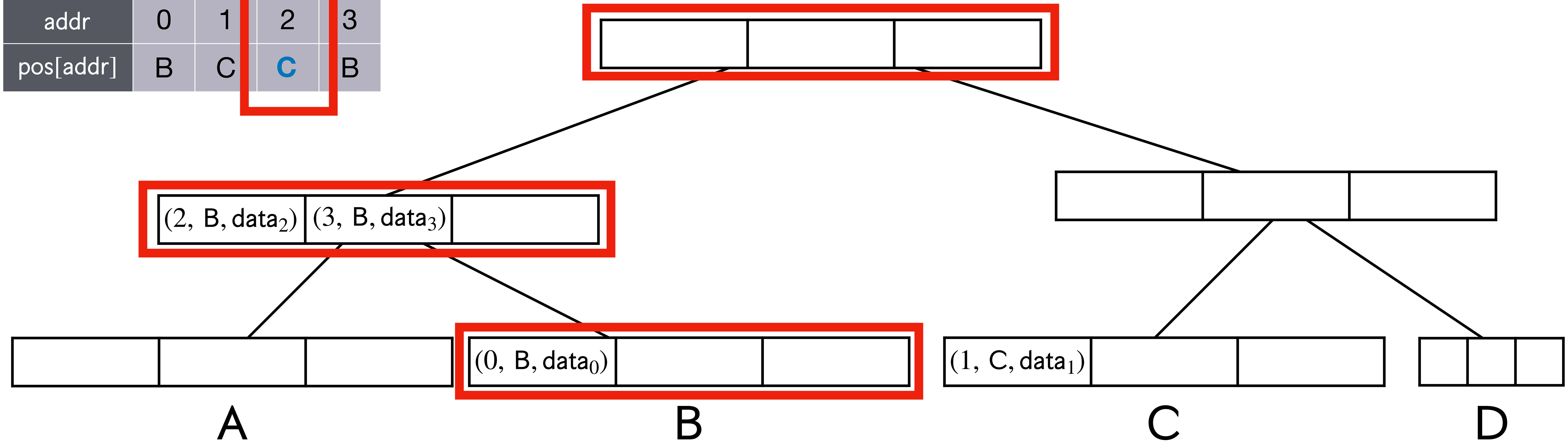

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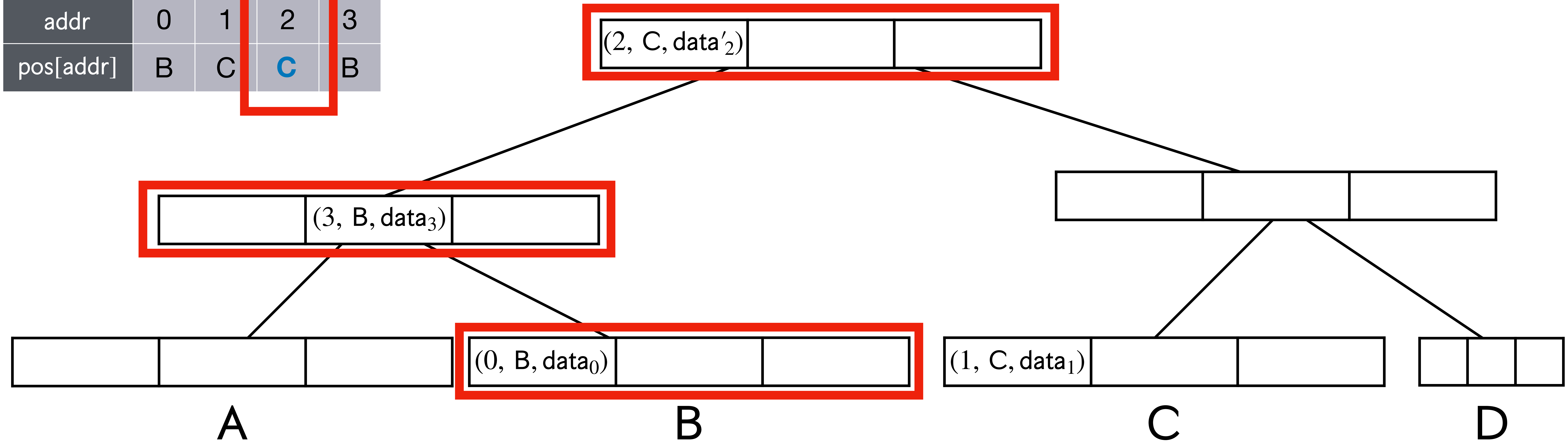

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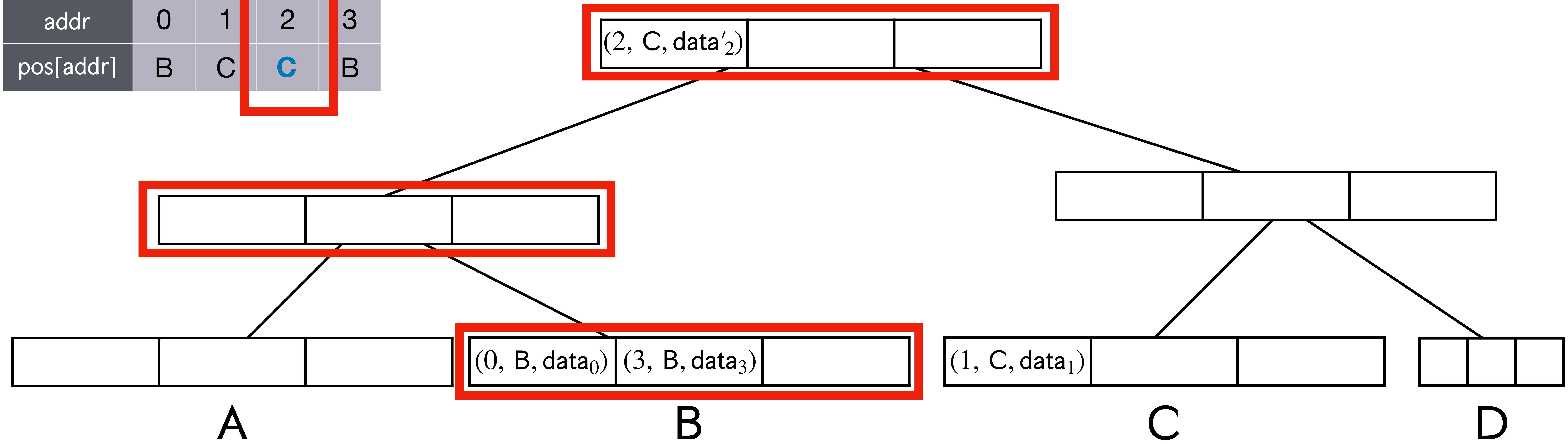

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  - (Technicality: also need to store $\omega(\log N)$-sized stash to prevent bucket overflow.)

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- Recently, they switched to using path ORAM instead, and they have seen a reduction from **500** servers to **6** servers!

Solving Privacy and Integrity Simultaneously: Maliciously Secure ORAM

Exercise

Show that Path ORAM is *not* maliciously secure, in the sense that a tampering adversary can **distinguish** between different user queries.

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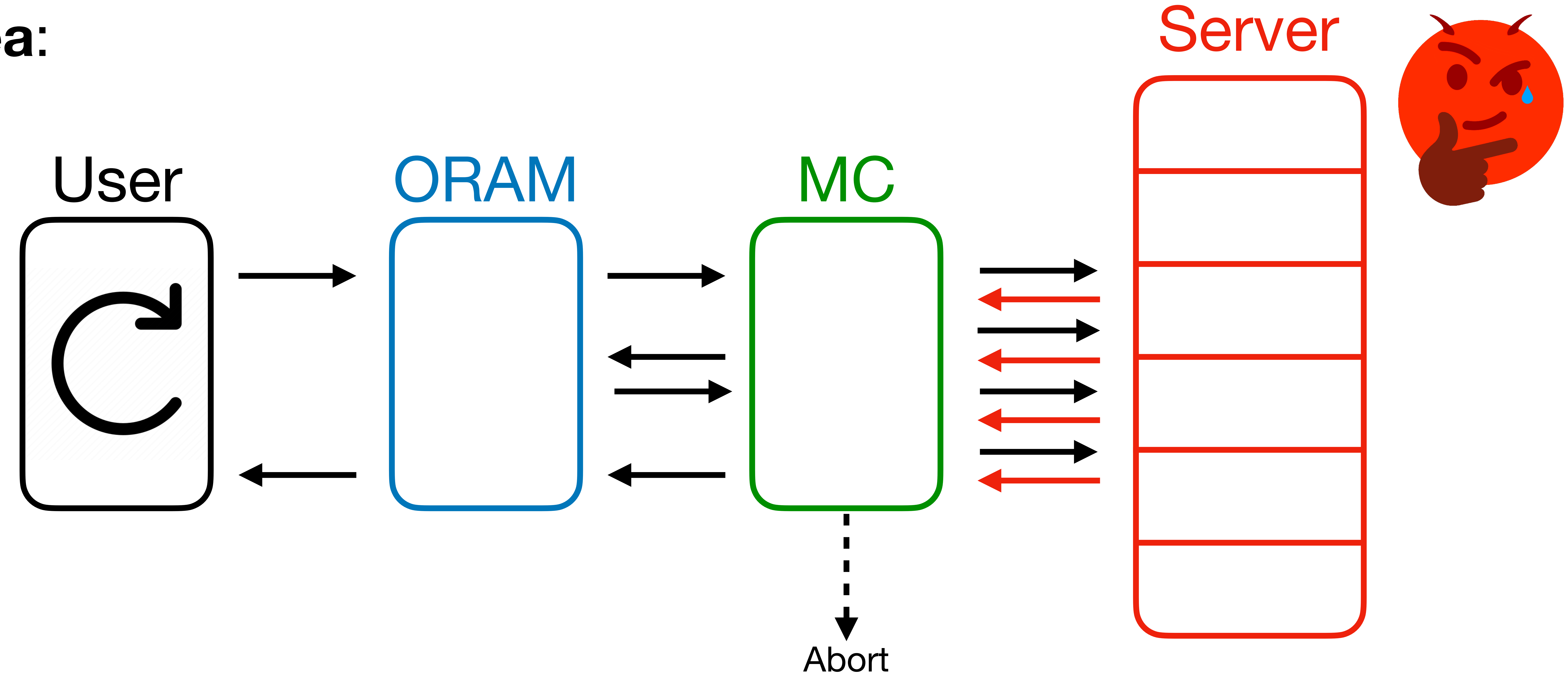

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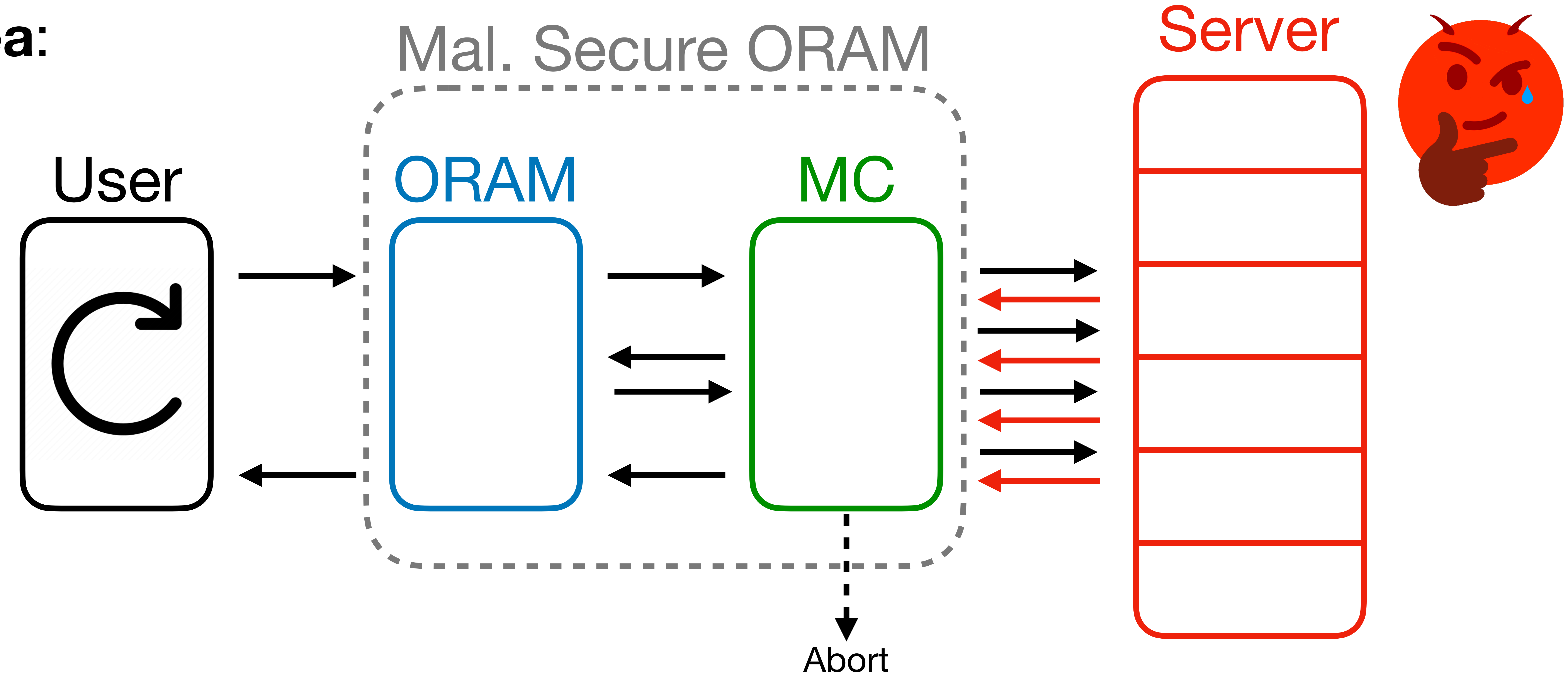

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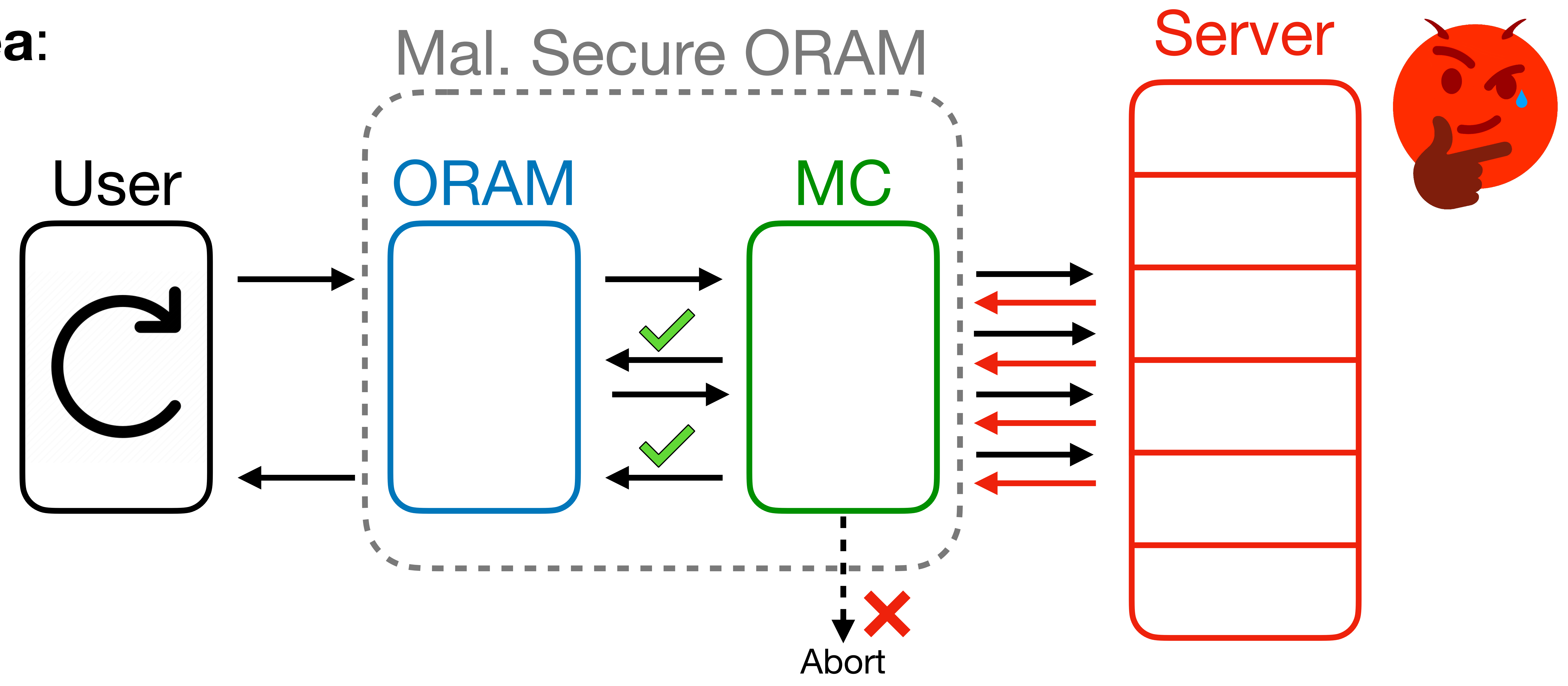

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- Can we non-trivially combine the two constructions we saw?

Yes!

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  - Specifically, for both constructions, each user query results in a lookup of the path from the root to the tree.
  - Run Path ORAM, and store and compute hashes along the way.
- **Result**: Maliciously secure ORAM with $O(\log^2 N)$ overhead!

Happy Thanksgiving!