MIT 6.875

Foundations of Cryptography
Lecture 13

Digital Signatures

We showed:

Theorem: Assuming the existence of one-way functions and collision-resistant hash function families, there are digital signature schemes.

Digital Signatures

It turns out that collision-resistant hashing is not necessary.

Theorem: Digital Signature schemes exist *if and only if* one-way functions exist.

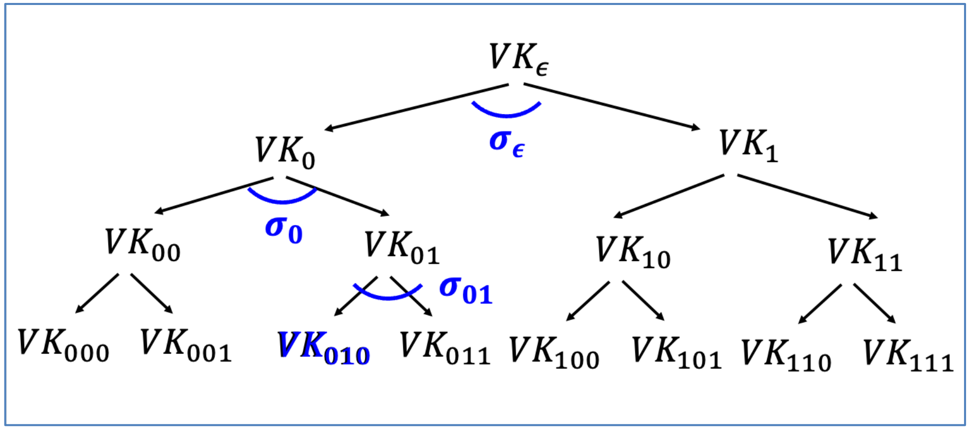
Digital Signature Construction

Start from $(OT.Gen, OT.Sign, OT.Ver)$, a one-time signature scheme that can sign arbitrarily long messages.

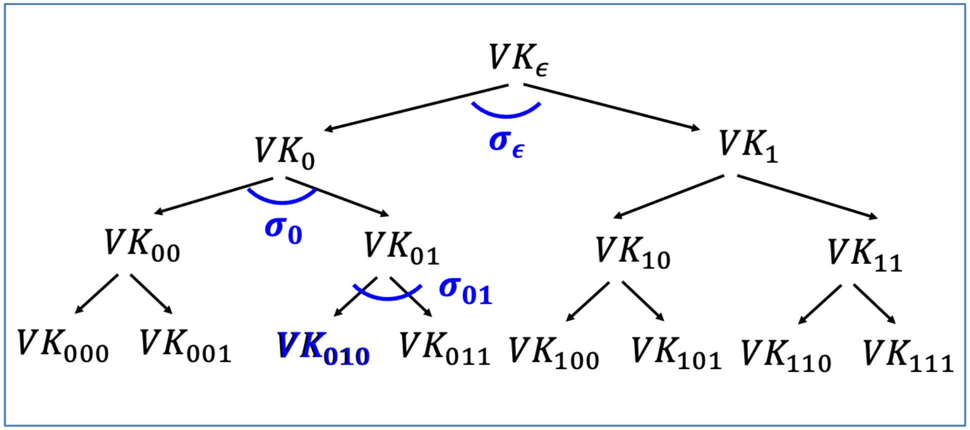
(Lamport + collision-resistant hashing)

Build a (virtual) tree of depth λ = security param.

Let K be a PRF key, $r_i = PRF(K, i)$ for $i \in \{0,1\}^{\leq \lambda}$, and $(VK_i, SK_i) \leftarrow OT.Gen(1^\lambda; r_i)$.



Digital Signature Construction



Signature keys: $SK = K$ and $VK = OTVK_\epsilon$.

Signing Algorithm:

Pick a random leaf $\mathrm{r} \in \{0,1\}^\lambda$,

Generate the authentication path $\sigma_\epsilon, \sigma_{r_1}, \sigma_{r_2}, \ldots, \sigma_r$ & σ^*

$$\sigma_x \leftarrow OT.Sign(SK_x, VK_{x0}||VK_{x1})$$

$$\sigma^* \leftarrow OT.Sign(SK_r, m)$$

The signature is $(\mathrm{r}, \sigma_\epsilon, \sigma_{r_1}, \sigma_{r_2}, \ldots, \sigma_r, \sigma^*)$.

Digital Signature Construction

- Historically regarded as inefficient; therefore, never used in practice.

- However, this signature scheme (or variants thereof) are now called “hash-based signatures” and seeing a re-emergence as a candidate post-quantum secure signature scheme. E.g. https://sphincs.org/

Direct Constructions

“Hash-and-Sign”: Secure in the “random oracle model”.

“Vanilla” RSA Signatures

Start with any trapdoor permutation, e.g. RSA.

Gen(1^λ): Pick primes (P, Q) and let $N = PQ$. Pick e relatively prime to $\varphi(N)$ and let $d = e^{-1} \pmod{\varphi(N)}$.

SK = (N, d) and VK = (N, e)

Sign(SK, m): Output signature $\sigma = m^d \pmod{N}$.

Verify(VK, m, σ): Check if $\sigma^e = m \pmod{N}$.

Problem: Existentially forgeable!

“Vanilla” RSA Signatures

Sign(SK, m): Output signature $\sigma = m^d \pmod{N}$.

Verify(VK, m, σ): Check if $\sigma^e = m \pmod{N}$.

Problem: Existentially forgeable!

Attack: Pick a random σ and output ($m = \sigma^e, \sigma$) as the forgery.

Problem: Malleable!

Attack: Given a signature of m, you can produce a signature of $2^e * m, 3^e *$ ^{3}, ...

“Vanilla” RSA Signatures

Sign(SK, m): Output signature $\sigma = m^d \pmod N$.

Verify($\mathrm{V}K, m, \sigma$): Check if $\sigma^e = m \pmod N$.

Fundamental Issues:

1. Can ”reverse-engineer” the message starting from the signature (Attack 1)

2. Algebraic structure allows malleability (Attack 2)

How to Fix Vanilla RSA

Start with any trapdoor permutation, e.g. RSA.

Gen(1^λ): Pick primes (P, Q) and let $N = PQ$. Pick e relatively prime to $\varphi(N)$ and let $d = e^{-1} \pmod{\varphi(N)}$.

SK = (N, d) and VK = $(N, e, \boldsymbol{H})$

Sign(SK, m): Output signature $\sigma = \boldsymbol{H}(\boldsymbol{m})^d \pmod{N}$.

Verify(VK, m, σ): Check if $\sigma^e = \boldsymbol{H}(\boldsymbol{m}) \pmod{N}$.

So, what is H? Some very complicated "hash" function.

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Start with any trapdoor permutation, e.g. RSA.

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SK = (N, d) and VK = $(N, e, \boldsymbol{H})$

Sign(SK, m): Output signature $\sigma = \boldsymbol{H(m)}^d \pmod{N}$.

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H should be at least one-way to prevent Attack #1.

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Hard to "algebraically manipulate" H(m) into H(related m').

(to prevent Attack #2.)

How to Fix Vanilla RSA

Start with any trapdoor permutation, e.g. RSA.

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Sign(SK, m): Output signature $\sigma = \boldsymbol{H(m)}^d \pmod{N}$.

Verify(VK, m, σ): Check if $\sigma^e = \boldsymbol{H(m)} \pmod{N}$.

Collision-resistance does not seem to be enough. (Given a CRHF h(m), you may be able to produce h(m’) for related m’.)

The Random Oracle Heuristic

Want: A public H that is “non-malleable”.

Given H(m), it is hard to produce H(m’) any *non-trivially related* m’.

For every PPT adv A and “every non-trivial relation” R,

$$\Pr\left[A\big(H(m)\big) = H(m'): R(m, m') = 1\right] = \mathrm{negl}(\lambda)$$

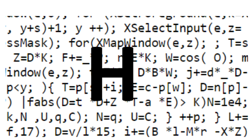
How about the relation R where
$R(x, y) = 1$ if and only if $y = \mathrm{H}(\mathrm{x})$?

The Random Oracle Heuristic

Proxy: A public H that "behaves like a random function"

(A PRF also behaves like a random function, but PRF_K is ***not*** publicly computable.)

Reality:

$\mathcal{A}($ 

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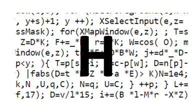
Random Oracle Heuristic:

$\mathcal{A}$ 

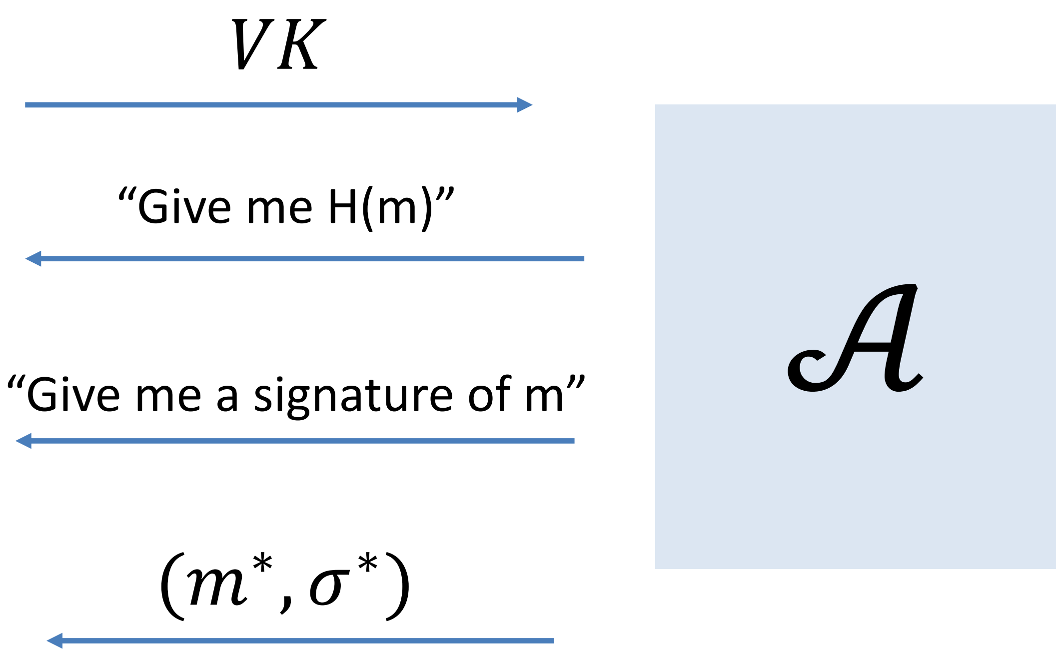
(1^λ)

The only way to compute H is by calling the oracle.

H is virtually a black box.

Proof

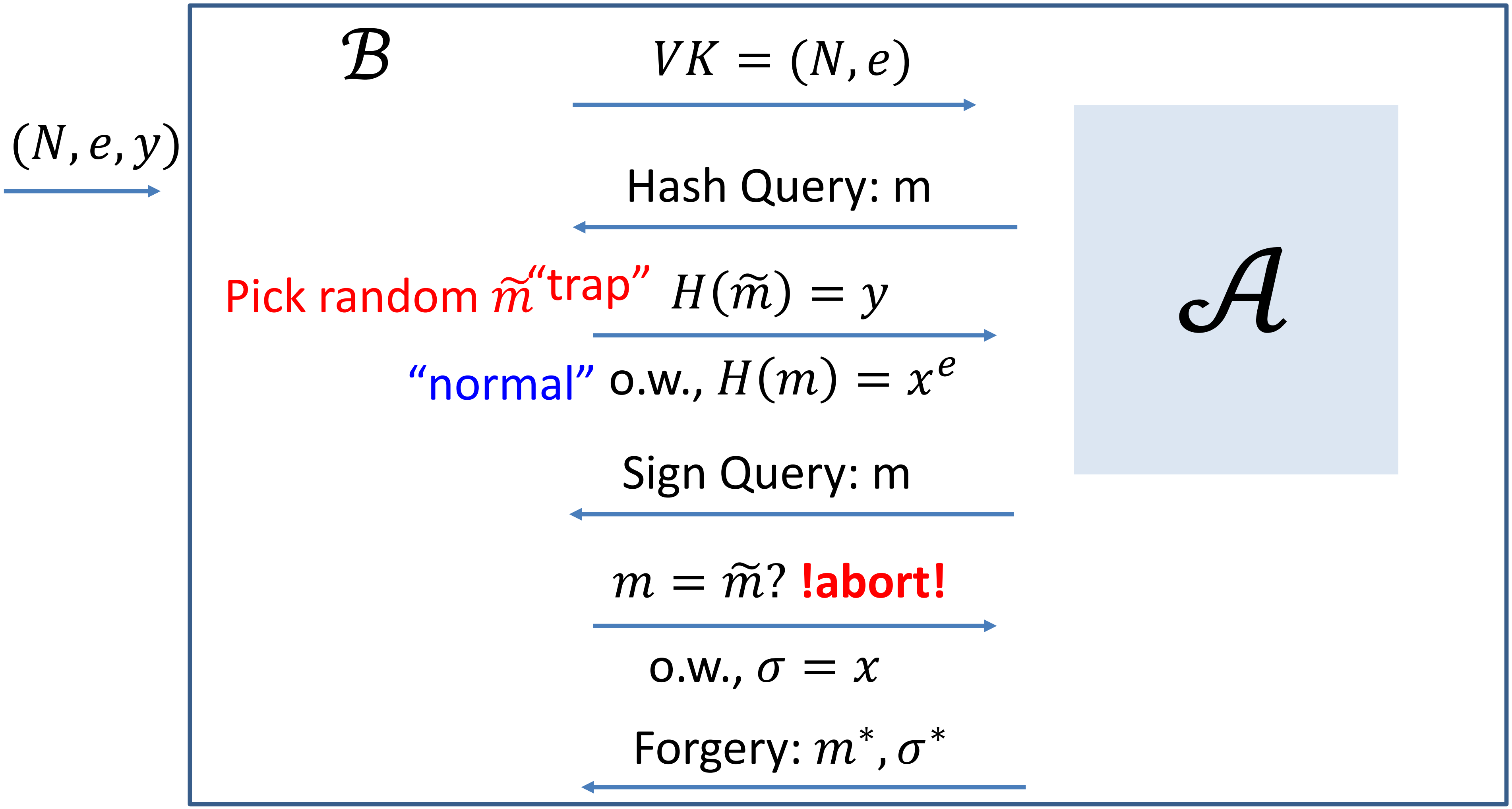
Assume there is a PPT adversary $\mathcal{A}$ that breaks the EUF-CMA security of hashed RSA in the random oracle model.



Then, there is an algorithm $\mathcal{B}$ that solves the RSA problem.

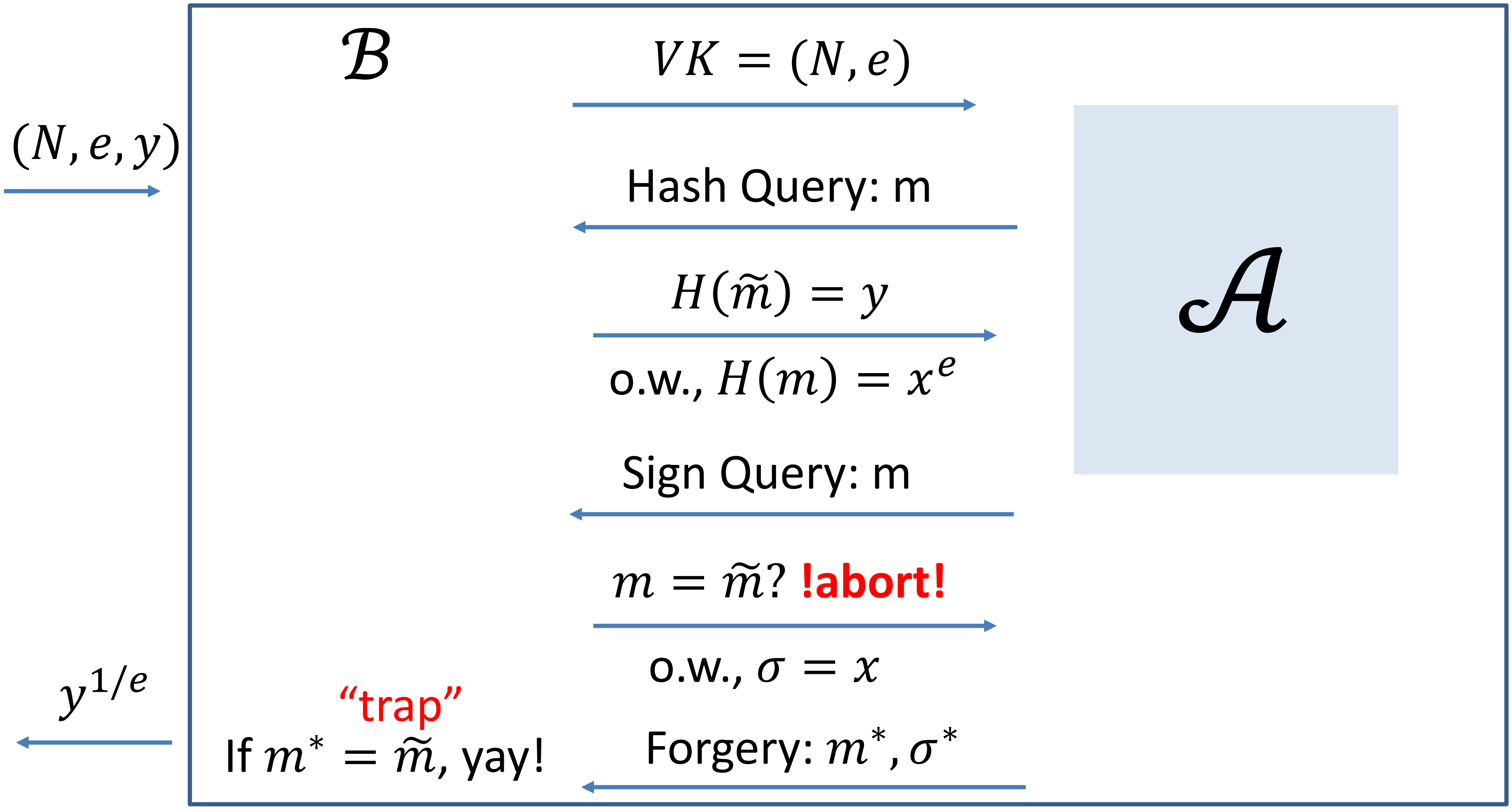
Proof

Assume there is a (Q-query) PPT adversary $\mathcal{A}$ that breaks the EUF-CMA security of hashed RSA in the random oracle model.



Proof

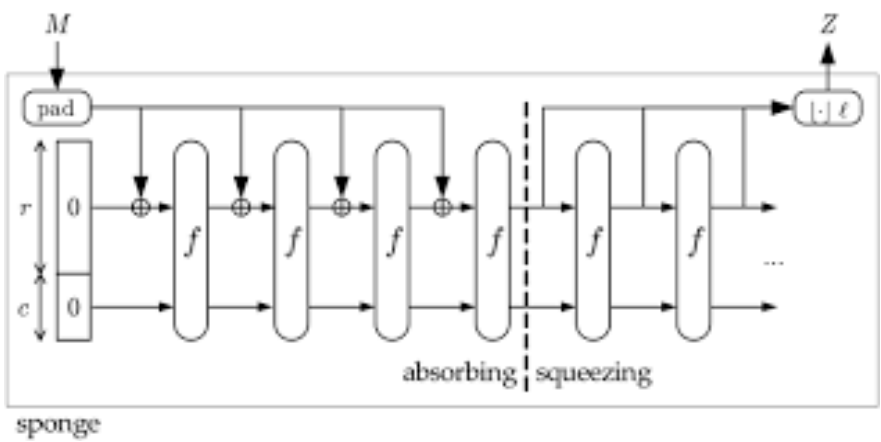
Claim: To produce a successful forgery, $\mathcal{A}$ must have queried the hash oracle on m^*. W.p. $1/Q$, m^* is the trap.



Bottomline: Hashed RSA

(PKCS Standard, used everywhere)

In practice, we let H be the SHA-3 hash function.



… and believe that SHA-3 ”acts like a random function”. That’s the heuristic. On the one hand, it doesn’t make any sense, but on the other, it has served us well so far. No attacks against RSA + SHA-3, for example.

Yet another signature scheme

Gennaro-Halevi-Rabin'99

Many Variants of Signatures

Aggregate Signatures: Compressing many signatures into one

Boneh-Lynn-Shacham (BLS) Signatures from "Bilinear maps"

Ring Signatures: Protection for Whistleblowers

Threshold Signatures: Protecting against loss of secret key

(won't show in this class)