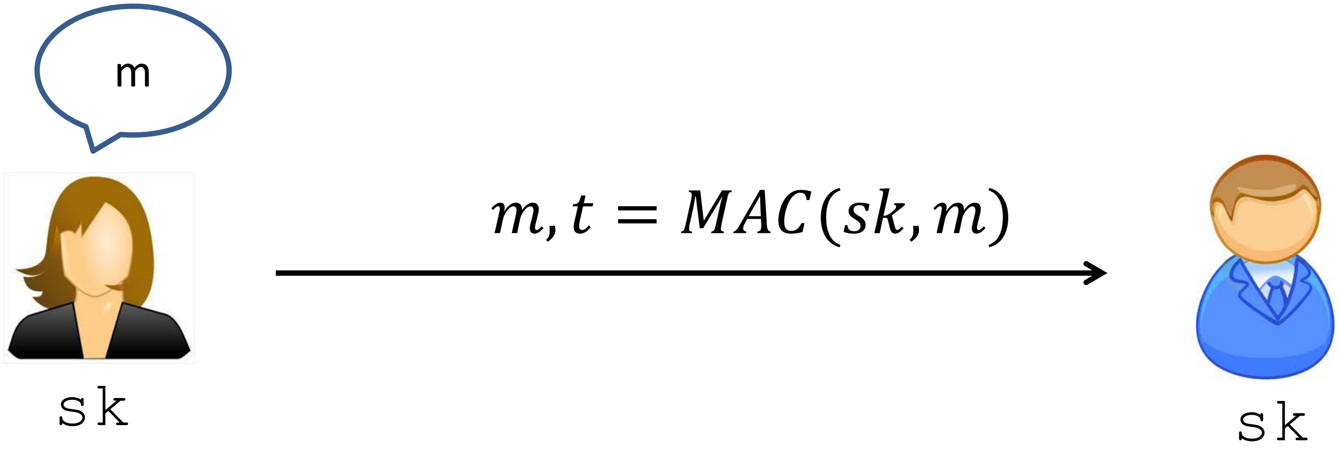
MIT 6.875

Foundations of Cryptography
Lecture 11

TODAY: Digital Signatures

Message Authentication Codes



m

$m, t = MAC(sk, m)$

sk

sk

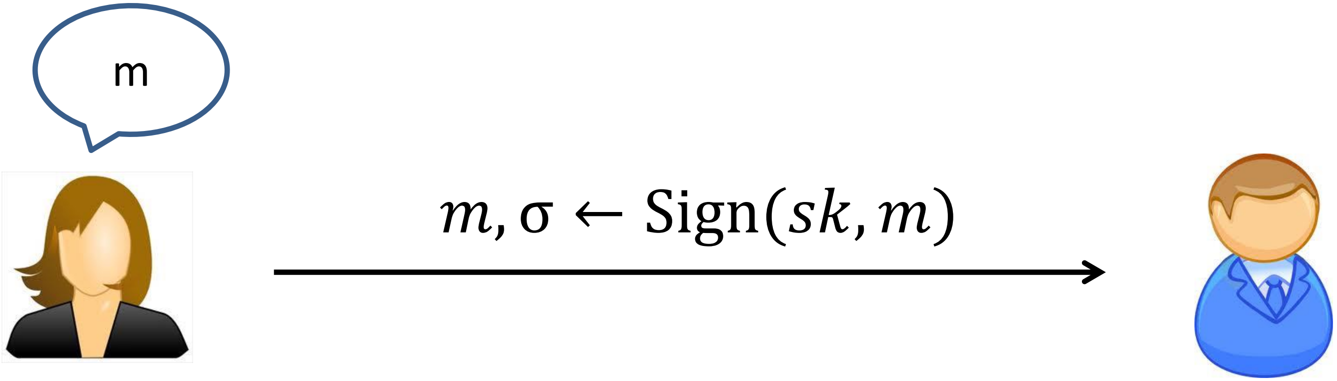
$\text{Verify}(sk, m, t)$

Authenticity: Bob wants to ensure that the message came from Alice.

Needs Bob and Alice to share a secret key beforehand.

Alice	**vk**

Digital Signatures: Public-key Analog of MACs



sk

$m, \sigma \leftarrow \mathrm{Sign}(sk, m)$

$\mathrm{Verify}(vk, m, \sigma)$

(Public) verification keys are stored in a “directory”.

Only Alice can produce signatures; but Bob (or indeed, anyone else) can verify them.

Digital Signatures vs. MACs

Signatures	**MACs**
n users require n key-pairs	n users require n^2 keys
Publicly Verifiable	Privately Verifiable
Transferable	**Not** Transferable
Provides Non-Repudiation (is this a good thing or a bad thing?)	Does not provide Non-Rep.

Alice	**pk,vk**

Other Applications

1. Certificates, or a public-key directory in practice:

Trusted Certificate Authority, e.g. Verisign, Let’s Encrypt.

When Alice *(=www.google.com)* wants to register her public (encryption and signing) keys pk and vk, first check that she *is* Alice.

Issue a “certificate” $\sigma \leftarrow Sign(SK_{Verisign}, Alice||pk||vk)$

Alice can later produce this certificate to prove that she “owns” pk and vk.

Browsers store $VK_{Verisign}$ and check the certificate.

Other Applications

2. Bitcoin and other cryptocurrencies:

I am identified by my verification key vk.

When I pay you (= vk'), I sign "\$x paid to vk'" with my sk.

Digital Signatures: Definition

A triple of PPT algorithms $(Gen, Sign, Verify)$ s.t.

- $(vk, sk) \leftarrow Gen(1^n)$.

PPT Key generation algorithm generates a public-private key pair.

- $\sigma \leftarrow Sign(sk, m)$.

(possibly probabilistic) Signing algorithm uses the secret signing key to produce a signature σ.

- $Acc(1)/Rej(0) \leftarrow Verify(vk, m, \sigma)$.

Verification algorithm uses the public verification key to check the signature σ against a message m.

Correctness: For all vk, sk, m:

$$Verify\big(vk, m, Sign(sk, m)\big) = \text{accept.}$$

Digital Signatures: Security

“The adversary after seeing signatures of many msgs, should not be able to produce a signature of any new msg.”

1. What are the adversary’s powers? Request for, and obtain, signatures of (poly many) messages $m_1, m_2, \ldots$

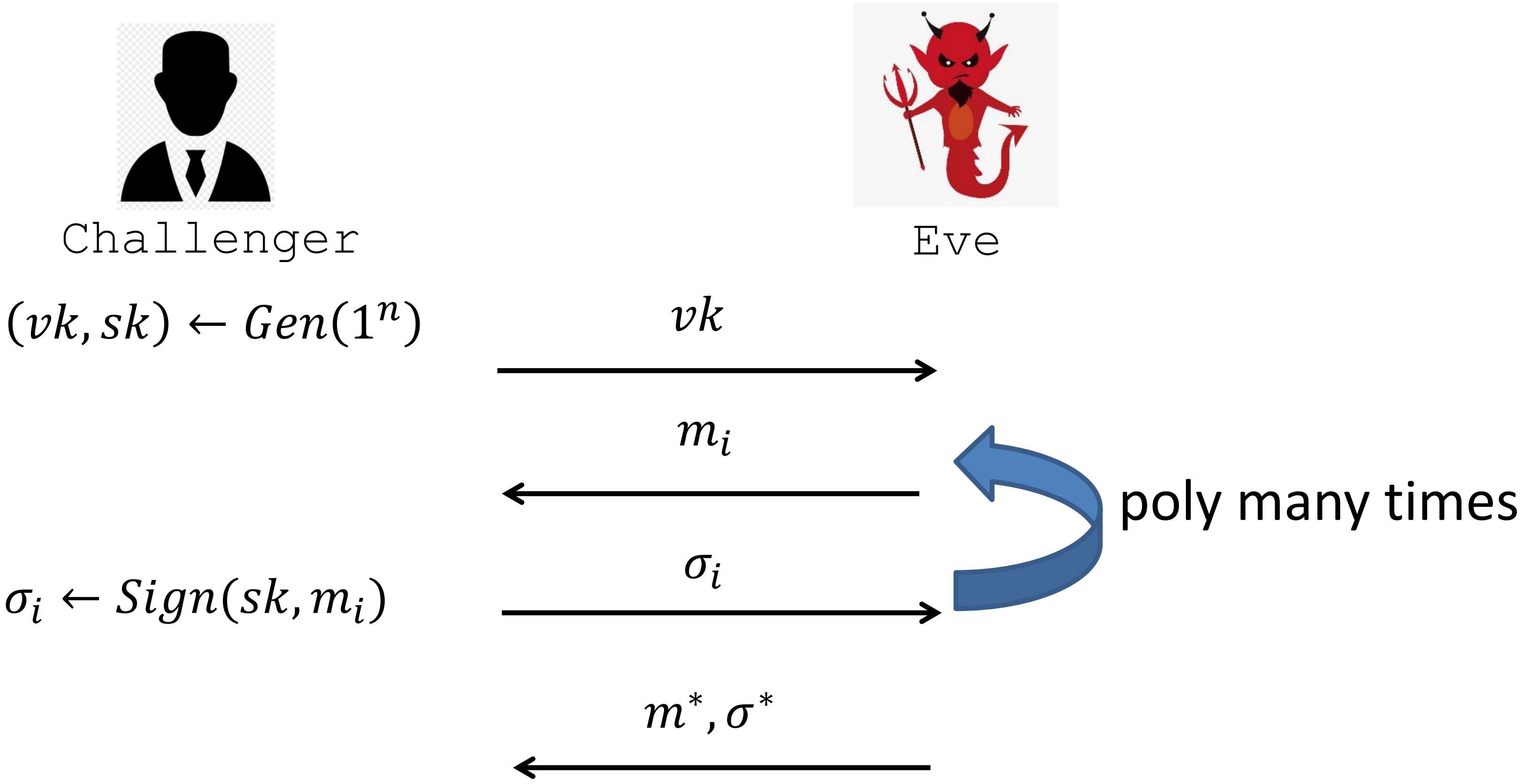
Chosen-message attack

2. What is her goal? She wins if she produces a signature of any message $m^* \notin \{m_1, m_2, \ldots\}$.

Existential Forgery

EUF-CMA Security

(Existentially Unforgeable against a Chosen Message Attack)



Challenger

Eve

$(vk, sk) \leftarrow Gen(1^n)$

vk

m_i

poly many times

σ_i

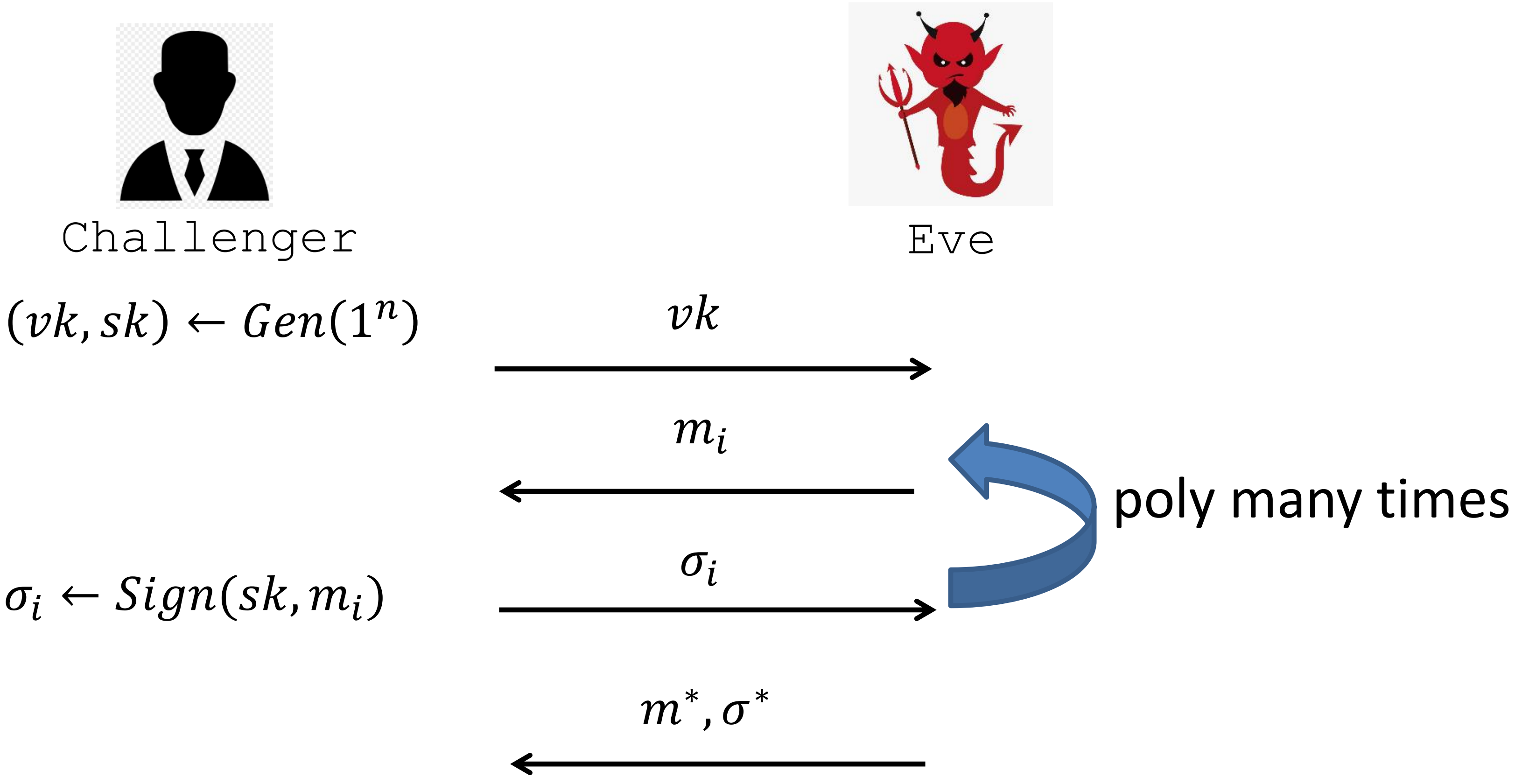
$\sigma_i \leftarrow Sign(sk, m_i)$

m^*, σ^*

Eve wins if Verify(vk, m^*, σ^*)$=1$ and $m^* \notin \{m_1, m_2, \ldots\}$.
The signature scheme is EUF-CMA-secure if no PPT Eve can win with probability better than negl(n).

Strong EUF-CMA Security

(Existentially Unforgeable against a Chosen Message Attack)



Challenger

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$\sigma_i \leftarrow Sign(sk, m_i)$

m^*, σ^*

Eve wins if Verify(vk, m^*, σ^*)= 1 and $\boldsymbol{(m^*, \sigma^*) \notin \{(m_1, \sigma_1), (m_2, \sigma_2), \ldots \}}$

The signature scheme is EUF-CMA-secure if no PPT Eve can win with probability better than negl(n).

Lamport (One-time) Signatures

How to sign a bit

Signing Key SK: $[x_0, x_1]$

Verification Key VK: $[y_0 = f(x_0), y_1 = f(x_1)]$

Signing a bit b: The signature is $\sigma = x_b$

Verifying (b, σ): Check if $f(\sigma) \overset{?}{=} y_b$

Claim: Assuming f is a OWF, no PPT adversary can produce a signature of $\bar{b}$ given a signature of b.

Lamport (One-time) Signatures

How to sign n bits

Signing Key SK: $\begin{bmatrix} x_{1,0} & x_{2,0} & & x_{n,0} \\ x_{1,1} & x_{2,1} & & x_{n,1} \end{bmatrix}$

Verification Key VK: $\begin{bmatrix} y_{1,0} & y_{2,0} & & y_{n,0} \\ y_{1,1} & y_{2,1} & & y_{n,1} \end{bmatrix}$

where $y_{i,c} = f(x_{i,c})$.

Signing an n-bit message $(m_1, \ldots, m_n)$:

The signature is $\left(x_{1,m_1}, \ldots, x_{n,m_n}\right)$.

Verifying $(\vec{m}, \vec{\sigma})$: Check if $\forall i$: $f(\sigma_i) \overset{?}{=} y_{i,m_i}$