What to Obfuscate When You're Obfuscating

Lecture 24

What is Obfuscation?

- **Program obfuscation** is a way to make a program unintelligible while *preserving functionality*.

What is Obfuscation?

- **Program obfuscation** is a way to make a program unintelligible while *preserving functionality*.

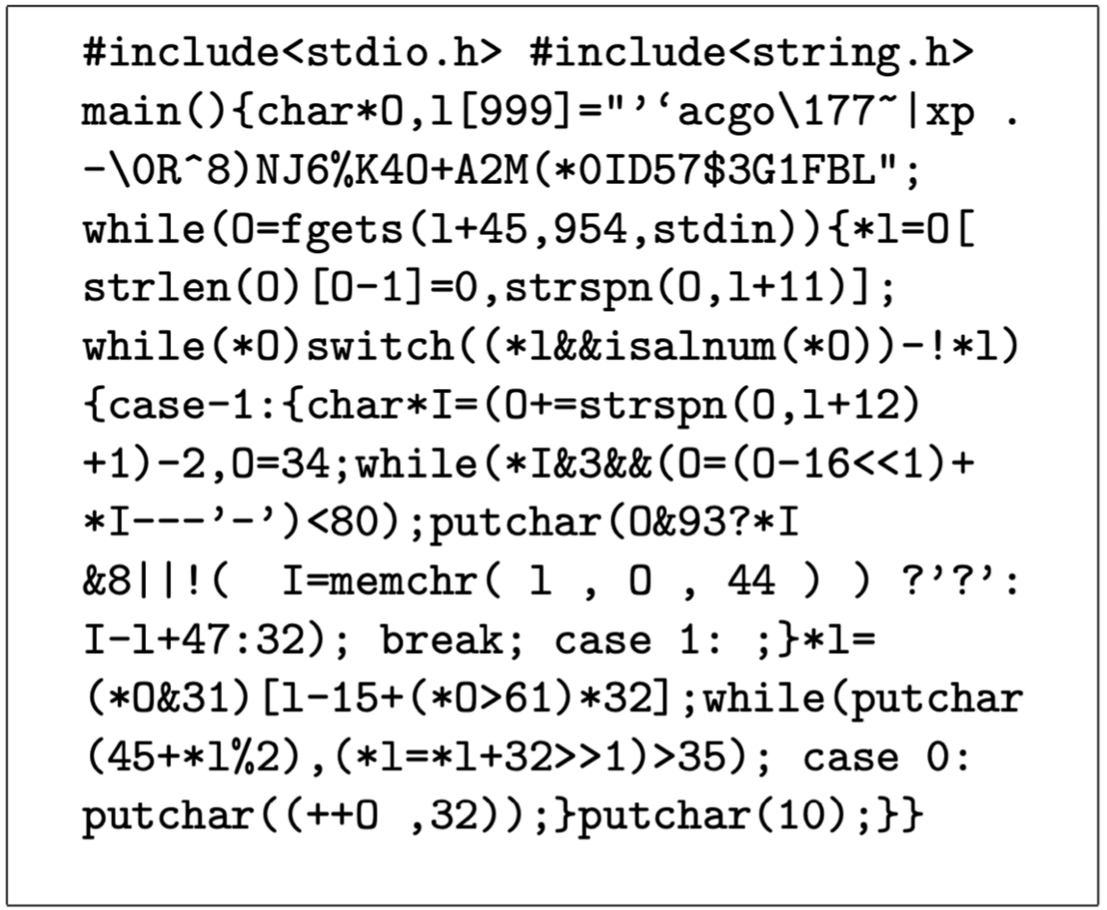
```
pi pi pi pi pi pi pi pi pi pi pika pipi pi pi pi pi pi pi pi pipi pi pi
pi pi pi pi pi pi pi pi pipi pi pi pi pipi pi pichu pichu pichu pichu ka
chu pipi pi pi pikachu pipi pi pikachu pi pi pi pi pi pi pi pikachu
pikachu pi pi pi pikachu pipi pi pi pikachu pichu pichu pi pi pi pi pi
pi pi pi pi pi pi pi pi pi pi pikachu pipi pikachu pi pi pi pikachu ka
ka ka ka ka ka pikachu ka ka ka ka ka ka ka ka pikachu pipi pi pikachu
pipi pikachu
```

What is Obfuscation?

- **Program obfuscation** is a way to make a program unintelligible while *preserving functionality*.

Hello, world! program

```
pi pi pi pi pi pi pi pi pi pi pika pipi pi pi pi pi pi pi pi pipi pi pi
pi pi pi pi pi pi pi pi pipi pi pi pi pipi pi pichu pichu pichu pichu ka
chu pipi pi pi pikachu pipi pi pikachu pi pi pi pi pi pi pi pikachu
pikachu pi pi pi pikachu pipi pi pi pikachu pichu pichu pi pi pi pi pi
pi pi pi pi pi pi pi pi pi pi pikachu pipi pikachu pi pi pi pikachu ka
ka ka ka ka ka pikachu ka ka ka ka ka ka ka ka pikachu pipi pi pikachu
pipi pikachu
```

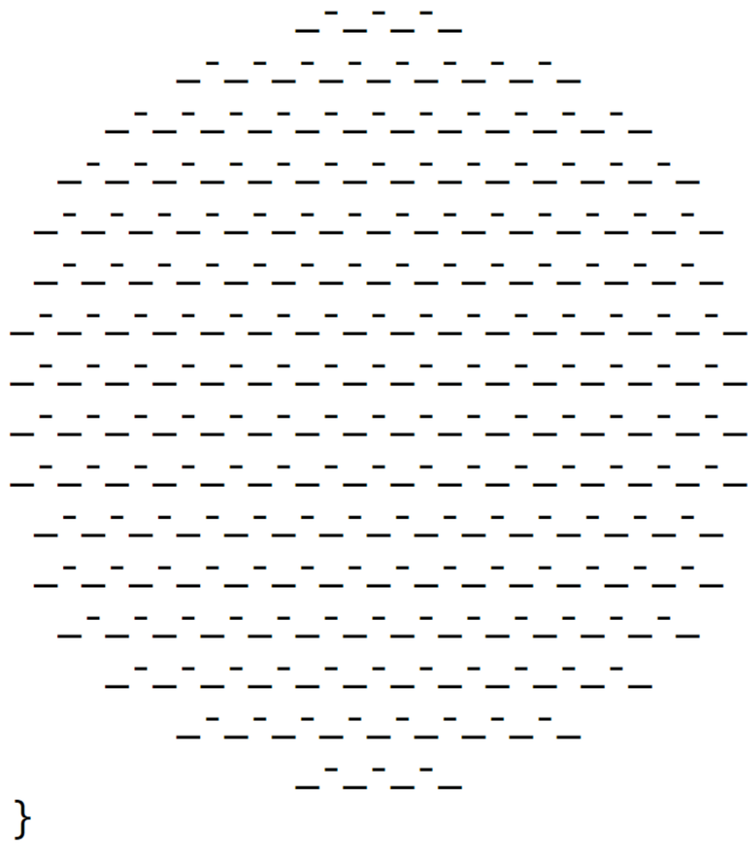


```
#include<stdio.h> #include<string.h>
main(){char*O,l[999]="’‘acgo\177~|xp .
-\OR^8)NJ6%K4O+A2M(*OID57$3G1FBL";
while(O=fgets(l+45,954,stdin)){*l=O[
strlen(O)[O-1]=0,strspn(O,l+11)];
while(*O)switch((*l&&isalnum(*O))-!*l)
{case-1:{char*I=(O+=strspn(O,l+12)
+1)-2,O=34;while(*I&3&&(O=(O-16<<1)+
*I---’-’)<80);putchar(O&93?*I
&8||!(  I=memchr( l , O , 44 ) ) ?’?’:
I-l+47:32); break; case 1: ;}*l=
(*O&31)[l-15+(*O>61)*32];while(putchar
(45+*l%2),(*l=*l+32>>1)>35); case 0:
putchar((++O ,32));}putchar(10);}}
```

Figure 1: The winning entry of the 1998 *International Obfuscated C Code Contest*, an ASCII/Morse code translator by Frans van Dorsselaer [vD] (adapted for this paper).

```
#include                                  <math.h>
#include                                 <sys/time.h>
#include                                 <X11/Xlib.h>
#include                                <X11/keysym.h>
                                        double L ,o ,P
                                       ,_=dt,T,Z,D=1,d,
                                       s[999],E,h= 8,I,
                                       J,K,w[999],M,m,O
                                      ,n[999],j=33e-3,i=
                                      1E3,r,t, u,v ,W,S=
                                      74.5,l=221,X=7.26,
                                      a,B,A=32.2,c, F,H;
                                      int N,q, C, y,p,U;
                                     Window z; char f[52]
                                   ; GC k; main(){ Display*e=
 XOpenDisplay( 0); z=RootWindow(e,0); for (XSetForeground(e,k=XCreateGC (e,z,0,0),BlackPixel(e,0))
; scanf("%lf%lf%lf",y +n,w+y, y+s)+1; y ++); XSelectInput(e,z= XCreateSimpleWindow(e,z,0,0,400,400,
0,0,WhitePixel(e,0) ),KeyPressMask); for(XMapWindow(e,z); ; T=sin(O)){ struct timeval G={ 0,dt*1e6}
; K= cos(j); N=1e4; M+= H*_; Z=D*K; F+=_*P; r=E*K; W=cos( O); m=K*W; H=K*T; O+=D*_*F/ K+d/K*E*_; B=
sin(j); a=B*T*D-E*W; XClearWindow(e,z); t=T*E+ D*B*W; j+=d*_*D-_*F*E; P=W*E*B-T*D; for (o+=(I=D*W+E
*T*B,E*d/K *B+v+B/K*F*D)*_; p<y; ){ T=p[s]+i; E=c-p[w]; D=n[p]-L; K=D*m-B*T-H*E; if(p [n]+w[ p]+p[s
]== 0|K <fabs(W=T*r-I*E +D*P) |fabs(D=t *D+Z *T-a *E)> K)N=1e4; else{ q=W/K *4E2+2e2; C= 2E2+4e2/ K
 *D; N-1E4&& XDrawLine(e ,z,k,N ,U,q,C); N=q; U=C; } ++p; } L+=_* (X*t +P*M+m*l); T=X*X+ l*l+M *M;
  XDrawString(e,z,k ,20,380,f,17); D=v/l*15; i+=(B *l-M*r -X*Z)*_; for(; XPending(e); u *=CS!=N){
                                  XEvent z; XNextEvent(e ,&z);
                                      ++*((N=XLookupKeysym
                                        (&z.xkey,0))-IT?
                                        N-LT? UP-N?& E:&
                                        J:& u: &h); --*(
                                        DN -N? N-DT ?N==
                                        RT?&u: & W:&h:&J
                                         ); } m=15*F/l;
                                         c+=(I=M/ l,l*H
                                         +I*M+a*X)*_; H
                                         =A*r+v*X-F*l+(
                                         E=.1+X*4.9/l,t
                                         =T*m/32-I*T/24
                                          )/S; K=F*M+(
                                          h* 1e4/l-(T+
                                          E*5*T*E)/3e2
                                          )/S-X*d-B*A;
                                          a=2.63 /l*d;
                                          X+=( d*l-T/S
                                           *(.19*E +a
                                           *.64+J/1e3
                                           )-M* v +A*
                                           Z)*_; l +=
                                           K *_; W=d;
                                           sprintf(f,
                                           "%5d  %3d"
                                           "%7d",p =l
                                           /1.7,(C=9E3+
                                O*57.3)%0550,(int)i); d+=T*(.45-14/l*
                               X-a*130-J* .14)*_/125e2+F*_*v; P=(T*(47
                               *I-m* 52+E*94 *D-t*.38+u*.21*E) /1e2+W*
                               179*v)/2312; select(p=0,0,0,0,&G); v-=(
                                W*F-T*(.63*m-I*.086+m*E*19-D*25-.11*u
                                 )/107e2)*_; D=cos(o); E=sin(o); } }
```

```
#define _ F-->00 || F-OO--;
long F=00,OO=00;
main(){F_OO();printf("%1.3f\n", 4.*-F/OO/OO);}F_OO()
{
```

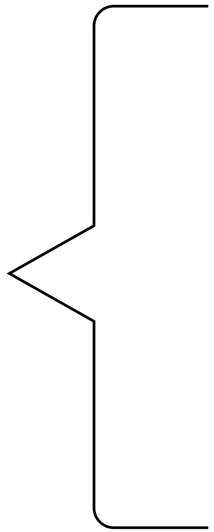


```
}
```

Answer: Run me!

Program Obfuscation

Programs that contain secrets:



Program Obfuscation

Programs that contain secrets:

- Cryptographic keys

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- Watermarks

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- Watermarks
- Trapdoors

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Programs that contain secrets:

- Cryptographic keys
- Watermarks
- Trapdoors
- The Algorithm itself

Hiding secrets

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Hmm… I am going out of town but would like to delegate all 6.875 emails to my TAs.

Hiding secrets

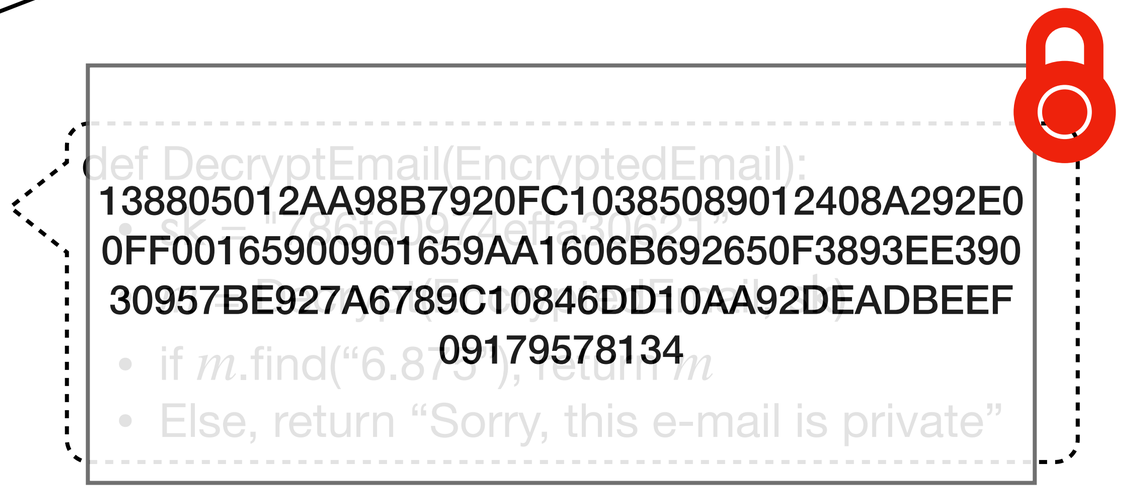
Hmm… I am going out of town but would like to delegate all 6.875 emails to my TAs.

def DecryptEmail(EncryptedEmail):

- sk = "786fe0974effa30621”
- m = Decrypt(EncryptedEmail, sk)
- if m.find(“6.875”), return m
- Else, return “Sorry, this e-mail is private”

Hiding secrets

Hmm… I am going out of town but would like to delegate all 6.875 emails to my TAs.



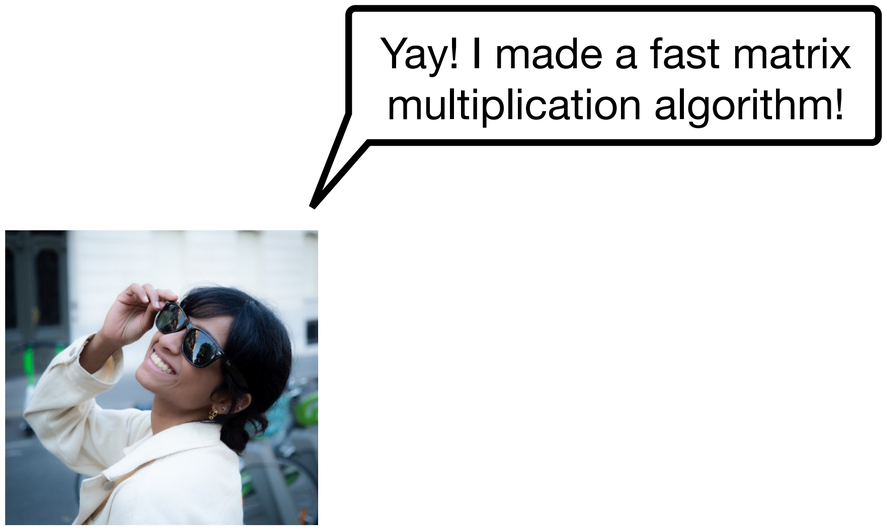
Hiding secrets

Hmm… I am going out of town but would like to delegate all 6.875 emails to my TAs.

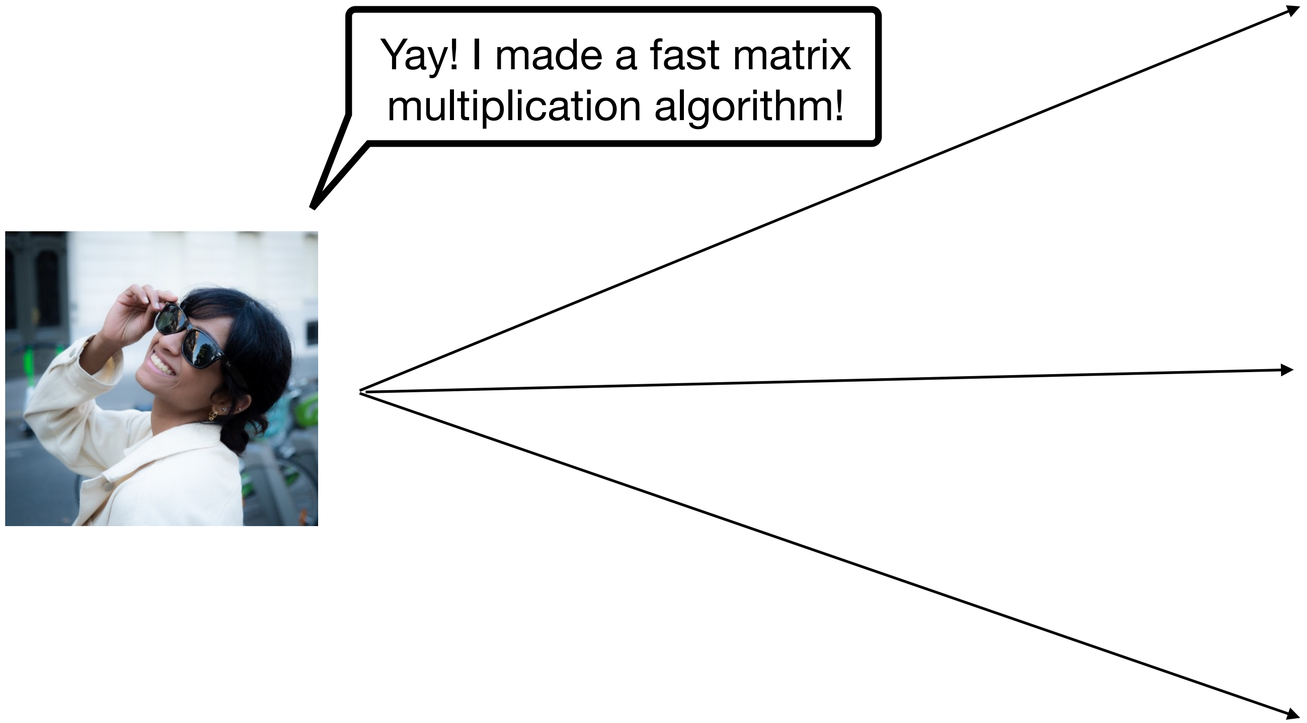
```
def DecryptEmail(EncryptedEmail):
    sk = "786fe0974effa3b621"
    m = Decrypt(sk, EncryptedEmail)
    if m.find("6.875"), return m
    Else, return "Sorry, this e-mail is private"
```

138805012AA98B7920FC10385089012408A292E0
0FF00165900901659AA1606B692650F3893EE390
30957BE927A6789C10846DD10AA92DEADBEEF
09179578134

Watermarking



Watermarking

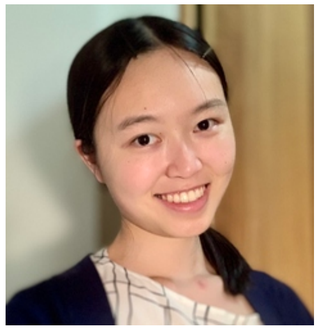


Watermarking

Yay! I made a fast matrix multiplication algorithm!

FMM
Author: Surya
Customer: Vinod

FMM
Author: Surya
Customer: Tina



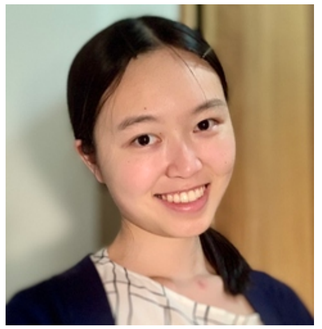
FMM
Author: Surya
Customer: Matt

Watermarking

Yay! I made a fast matrix multiplication algorithm!

3bcc4baa285258a4
242c4bd5092108fa
8ac7460be9a97706

6bf31c3e5aa0e434
46f98ceb880d6750
0a7be5d9807d11cf



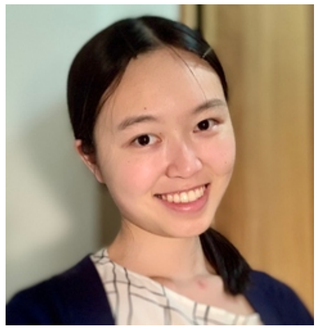
527031f92ffae4286
87521850702de15
da843c27062ee79c

Watermarking

Yay! I made a fast matrix multiplication algorithm!

3bcc4baa285258a4
242c4bd5092108fa
8ac7460be9a97706

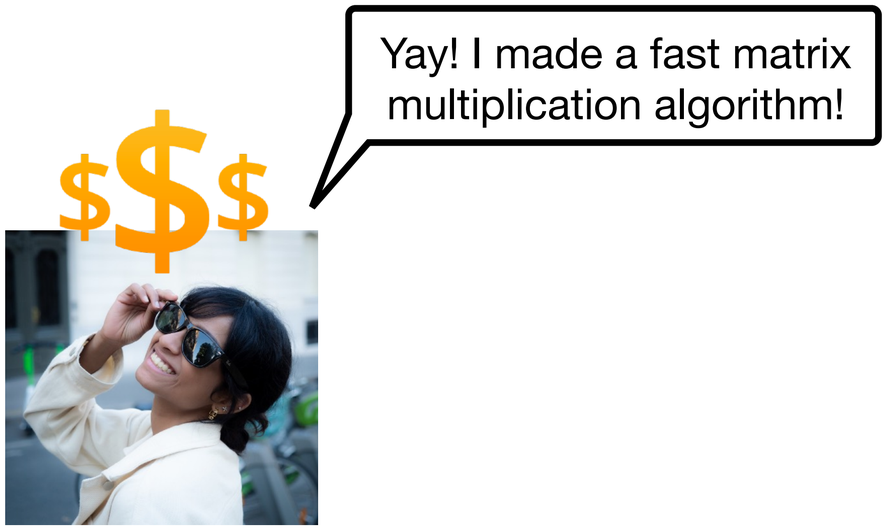
6bf31c3e5aa0e434
46f98ceb880d6750
0a7be5d9807d11cf



527031f92ffae4286
87521850702de15
da843c27062ee79c

The watermarks are
now difficult to remove!

The algorithm itself!



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$1 million

FMM
Runtime: $O(n^2)$

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Algorithm with $O(n^{2.5})$ dummy steps

$10k

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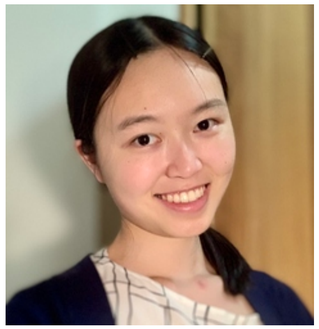

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Algorithm with $O(n^{2.5})$ dummy steps

Algorithm with $O(n^3)$ dummy steps

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$10k

FMM
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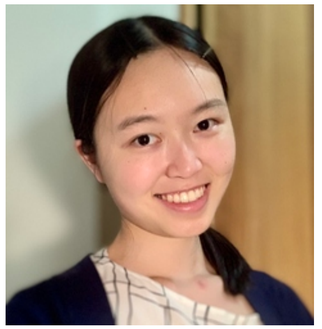

$1

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The algorithm itself!

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Algorithm with $O(n^{2.5})$ dummy steps

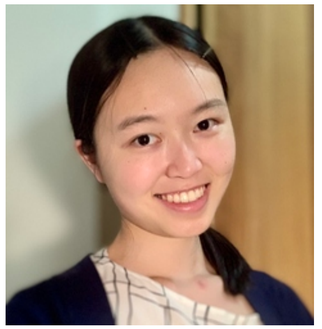
Algorithm with $O(n^3)$ dummy steps

$1 million

cdb443b0a41f518d
9ae55ae65a3c9c8d
ac37e34f63ad52f7a

$10k

088c2493182b421f
1b376c46909148cb
6a732c75ac75b989



$1

4ebaf73b13cd4d8d
c89bc0141b21d5c2
e9081ec6ff8a68f0d

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Algorithm with $O(n^{2.5})$ dummy steps

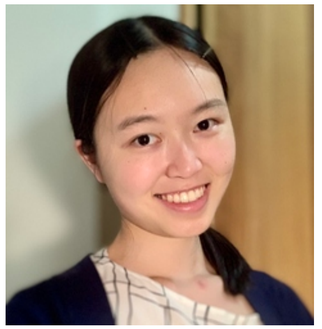
Algorithm with $O(n^3)$ dummy steps

$1 million

cdb443b0a41f518d
9ae55ae65a3c9c8d
ac37e34f63ad52f7a

$10k

088c2493182b421f
1b376c46909148cb
6a732c75ac75b989



$1

4ebaf73b13cd4d8d
c89bc0141b21d5c2
e9081ec6ff8a68f0d

This makes the fast algorithm hard to extract!

So… How do we define obfuscation?

Virtual Black-Box

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A ppt algorithm $\mathcal{O}$ is an obfuscation for a collection $\mathscr{C}$ of circuits if:

Virtual Black-Box

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- **(Perfect functionality)** $\Pr_r[\mathcal{O}(C; r) = C] = 1.$

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- **(Polynomial slowdown)** The size of $\mathcal{O}(C)$ of is poly($|C|$).

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A ppt algorithm $\mathcal{O}$ is an obfuscation for a collection $\mathcal{C}$ of circuits if:

- **(Perfect functionality)** $\Pr_r[\mathcal{O}(C; r) = C] = 1.$
- **(Polynomial slowdown)** The size of $\mathcal{O}(C)$ of is poly($|C|$).
- **(VBB property)** $\mathcal{O}(C)$ reveals no more information than black-box access to C!

Virtual Black-Box

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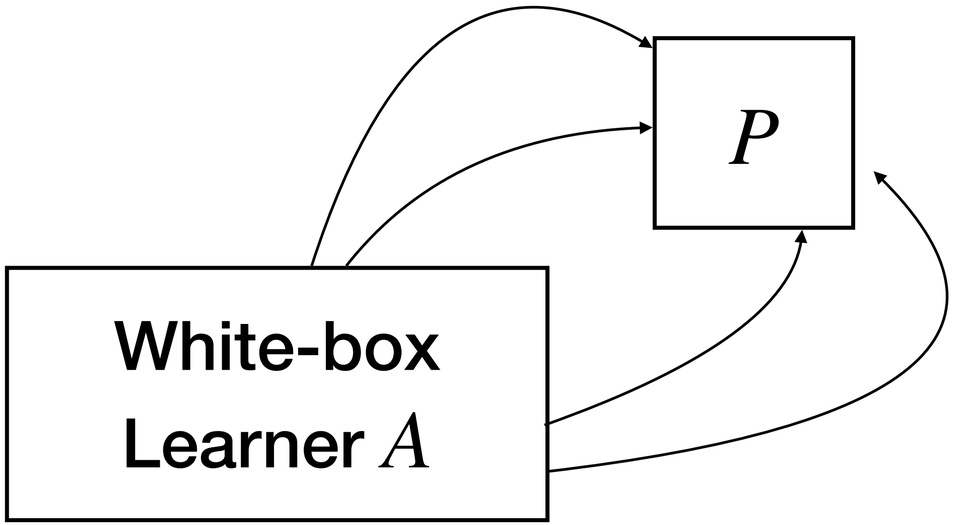
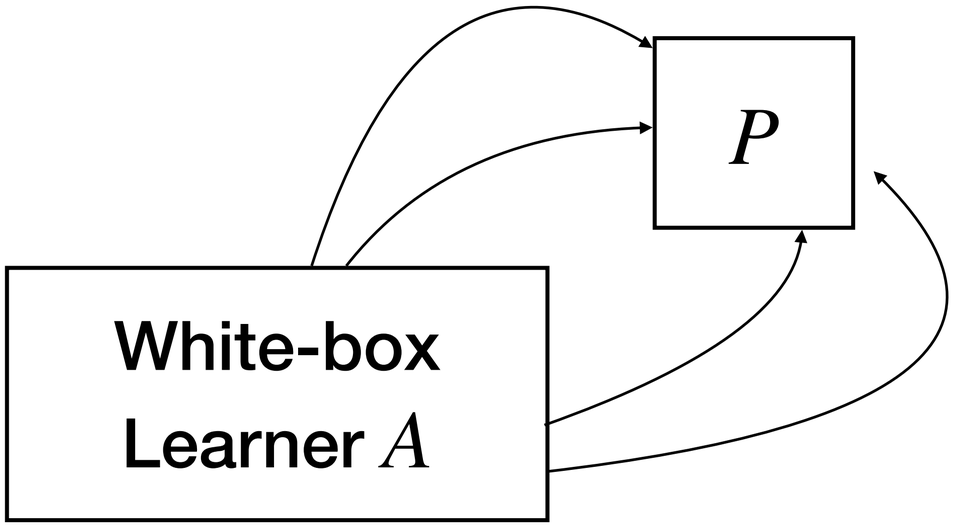
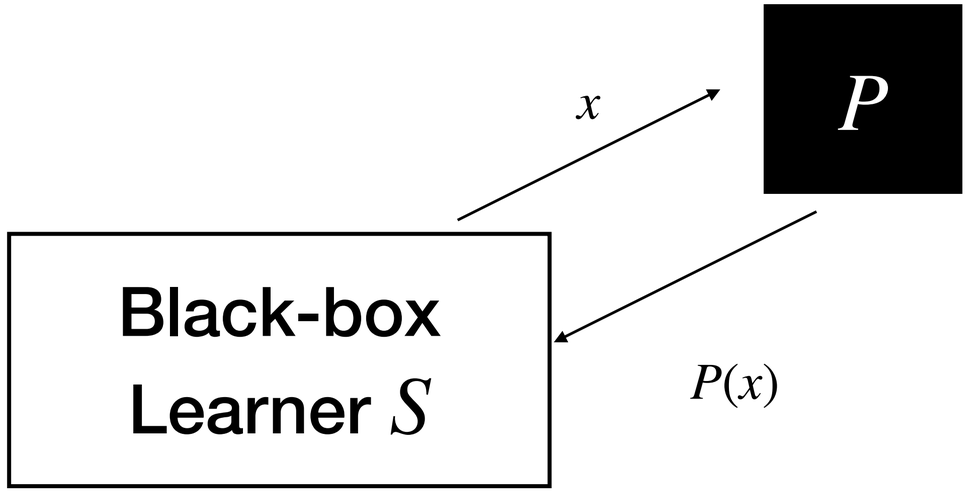


Virtual Black-Box

- $\mathcal{O}(C)$ reveals no more information than black-box access to C!





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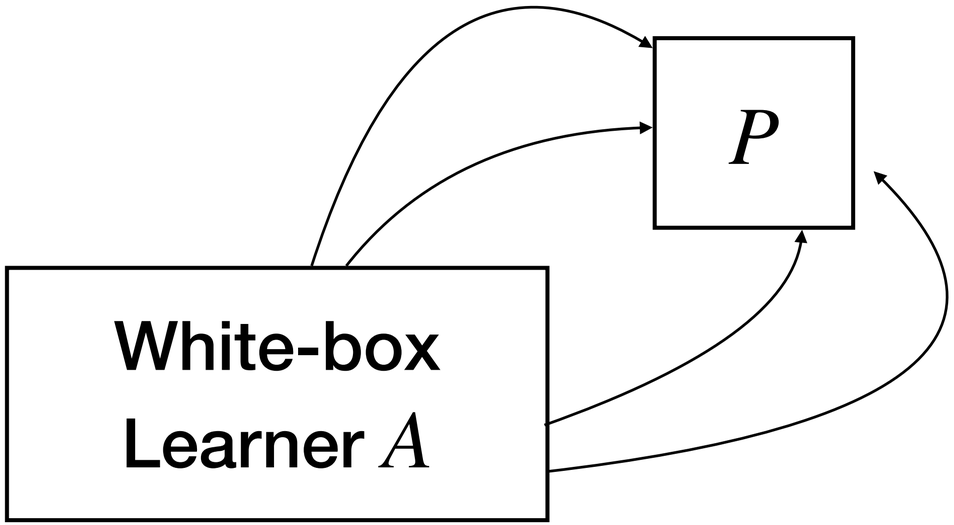
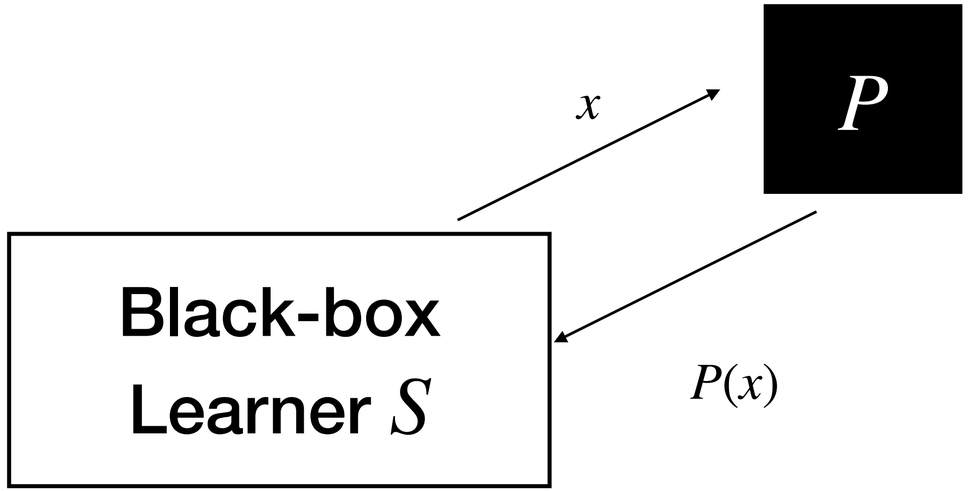


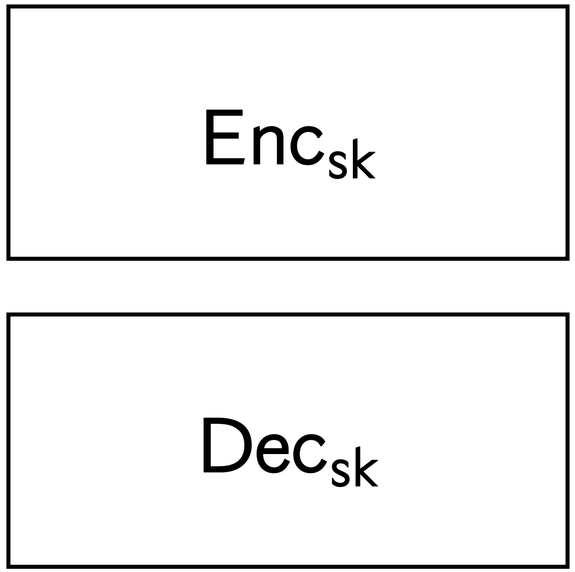


For all white-box learners A, there exists a simulator S such that

$$|\Pr[A(\mathcal{O}(C)) = 1] - \Pr[S^C(1^{|C|}) = 1]| \leq \mathsf{negl}(|C|)$$

PKE from SKE!

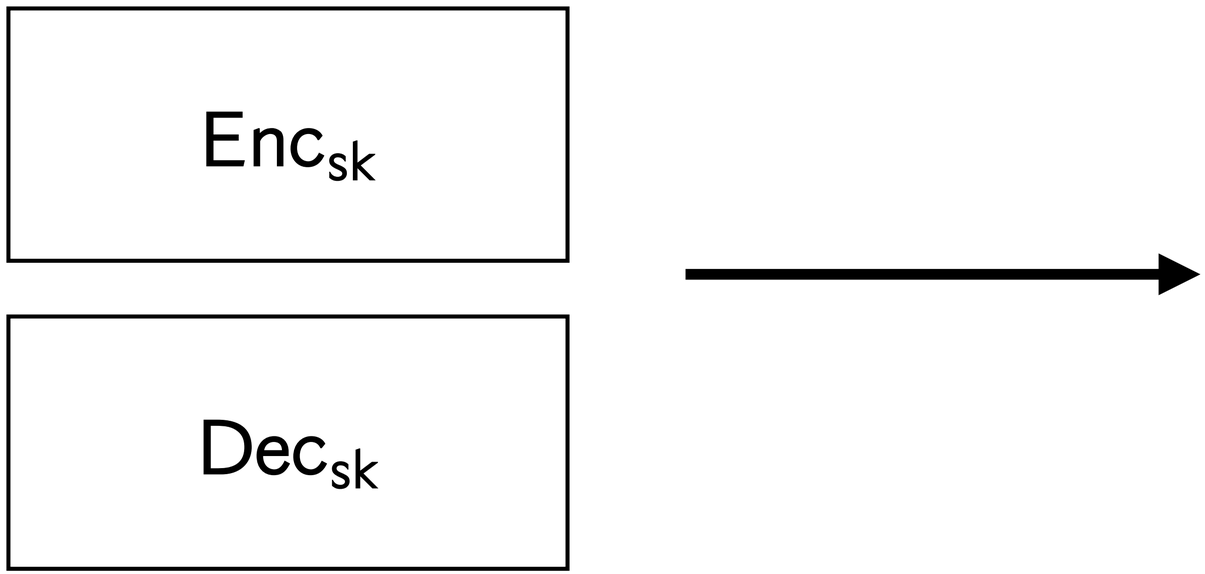
Secret-Key Encryption



Public-Key Encryption!

PKE from SKE!

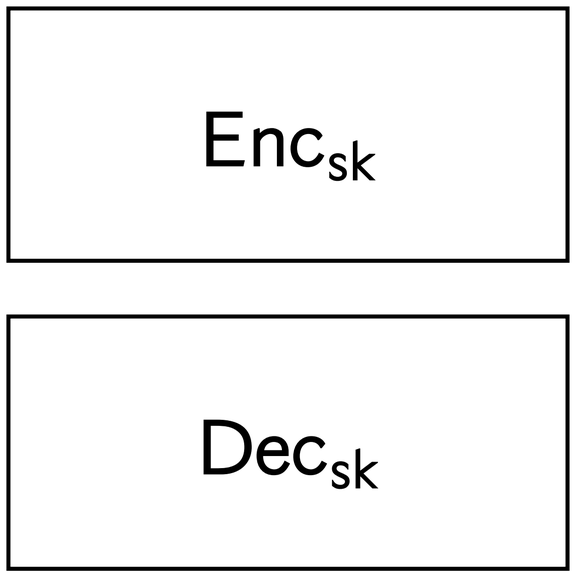
Secret-Key Encryption



Public-Key Encryption!

PKE from SKE!

Secret-Key Encryption

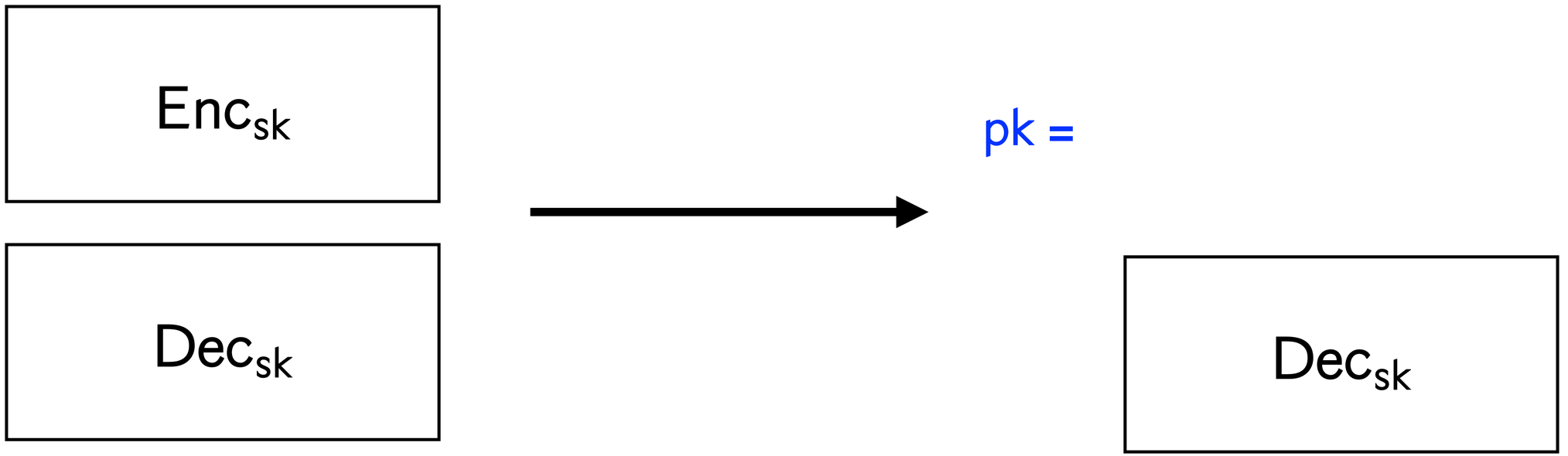


Public-Key Encryption!

Dec_{sk}

PKE from SKE!

Secret-Key Encryption



Public-Key Encryption!

PKE from SKE!

Secret-Key Encryption

Enc_{sk}

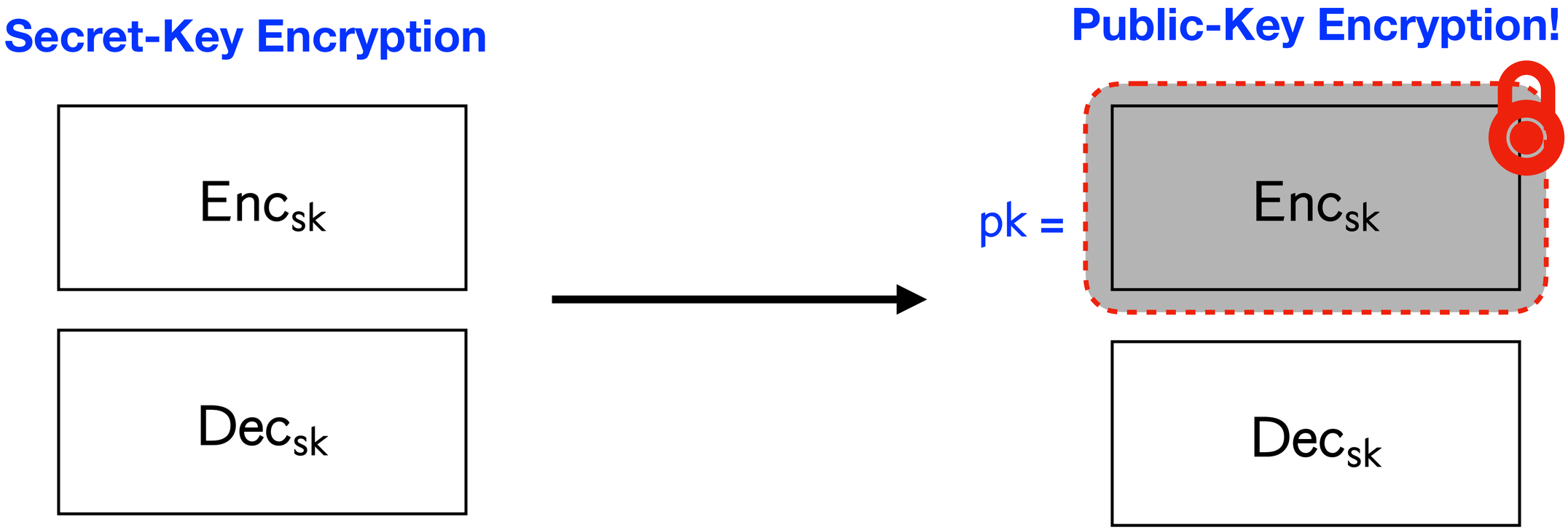
Dec_{sk}

Public-Key Encryption!

$pk =$ Enc_{sk}

Dec_{sk}

PKE from SKE!



OWF + VBB gives public key encryption!!

Note that we previously used LWE, DDH, RSA, QR etc. to obtain PKE from SKE.

Diffie-Hellman (1976)

Essentially what is required is a one-way compiler: one which takes an easily understood program written in a high level language and translates it into an incomprehensible program in some machine language. The compiler is one-way because it must be feasible to do the compilation, but infeasible to reverse the process. Since efficiency in size of program and run time are not crucial in this application, such compilers may be possible if the structure of the machine language can be optimized to assist in the confusion.

Fully-Homomorphic Encryption

OWF + VBB also gives you FHE!!

Fully-Homomorphic Encryption

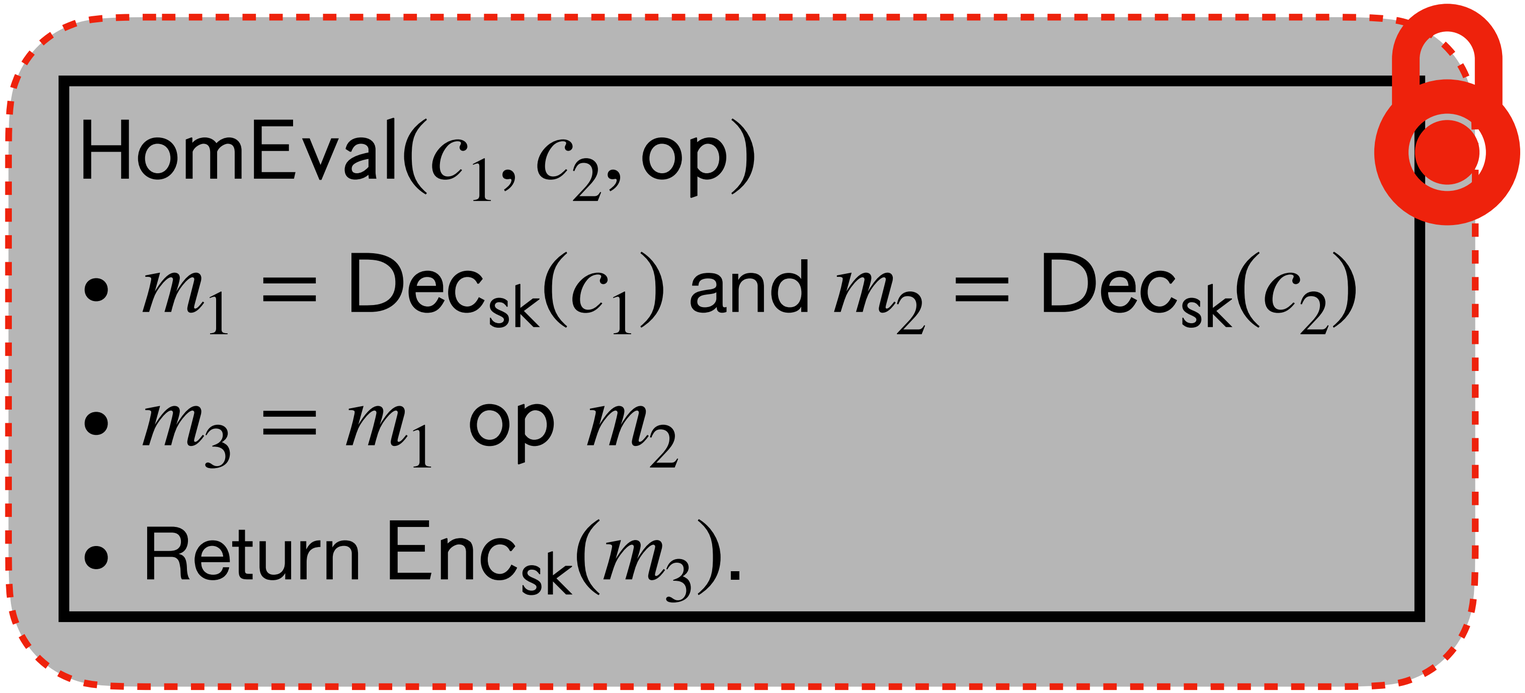
OWF + VBB also gives you FHE!!

HomEval(c_1, c_2, op)

- $m_1 = \mathsf{Dec}_{\mathsf{sk}}(c_1)$ and $m_2 = \mathsf{Dec}_{\mathsf{sk}}(c_2)$
- $m_3 = m_1$ op m_2
- Return $\mathsf{Enc}_{\mathsf{sk}}(m_3)$.

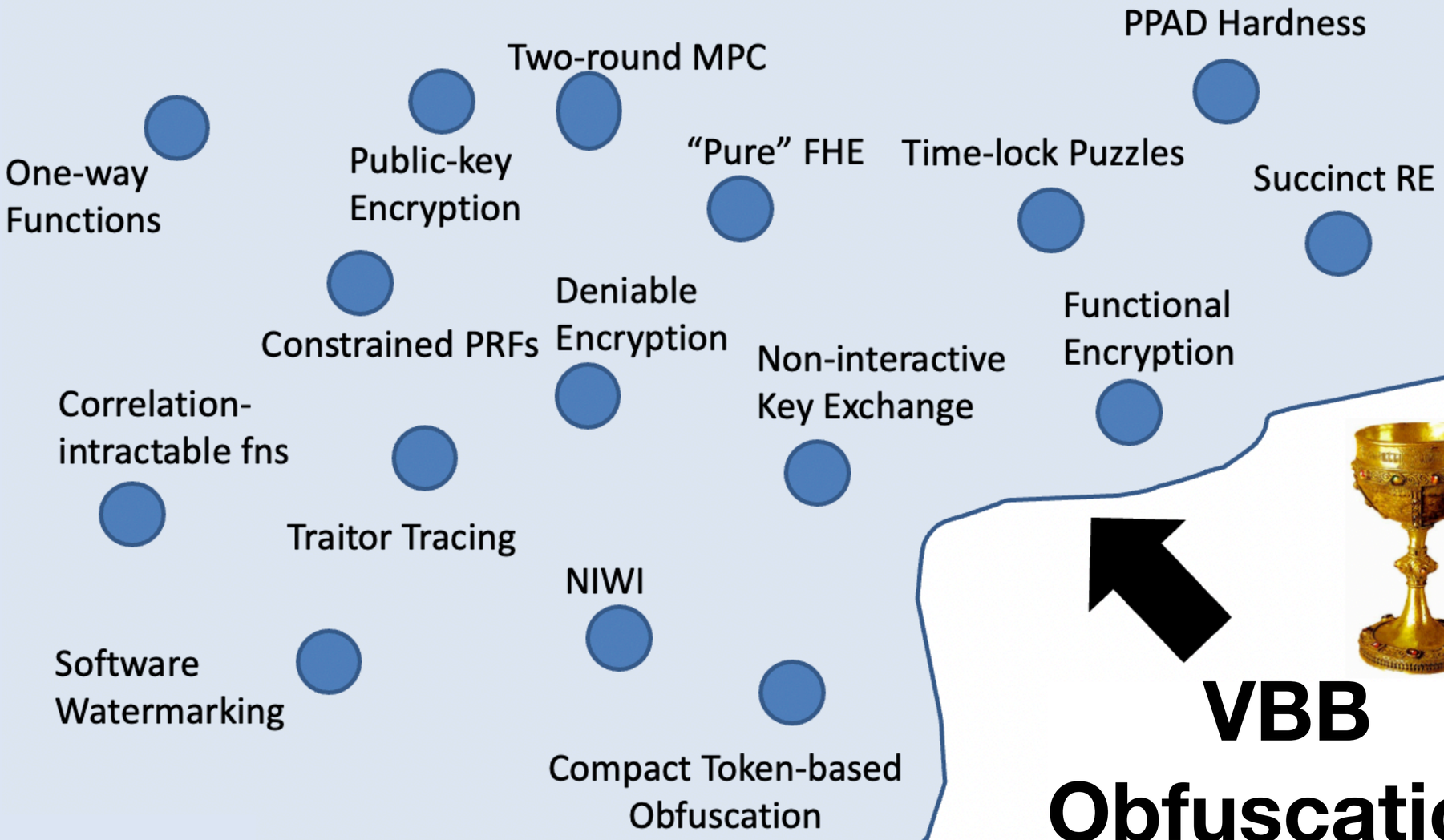
Fully-Homomorphic Encryption

OWF + VBB also gives you FHE!!



"CRYPTO-COMPLETE":

Nearly all crypto is an easy corollary of VBB!



Bad news…

Bad news…

Theorem [BGIRSVY ’01]. $\forall \mathcal{O}, \exists P$ such that $\mathcal{O}$ fails to VBB obfuscate P.

Bad news…

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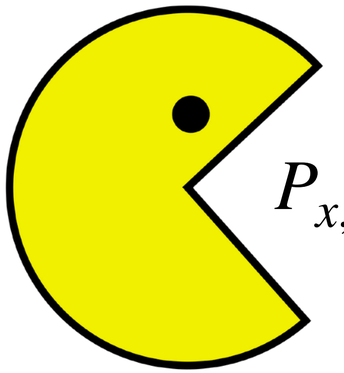

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Remarks

- **One interpretation:** Black-box security cannot be obtained if we give “physical code”
- Even inefficient VBB is impossible!
- Can be extended to construct unobfuscatable encryption/signature schemes.

Q: Uhm… What now?

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A: Weaken the definition!!

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Compare iO vs VBB

Virtual Black-Box Obfuscation

- **(Perfect functionality)**
  $\Pr_r[\mathcal{O}(C; r) = C] = 1.$
- **(Polynomial slowdown)**
  The size of $\mathcal{O}(C)$ is poly($|C|$).
- **(VBB property)**
  $\mathcal{O}(C)$ reveals no more information than black-box access to C!

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  The size of $i\mathcal{O}(C)$ is poly($|C|$).
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Remark 1: In fact, one can think of iO as a “pseudo-canonicaliser”.

Remark 2: This fact means that it is hard to show iO implies OWF (if it did, P ≠ NP).

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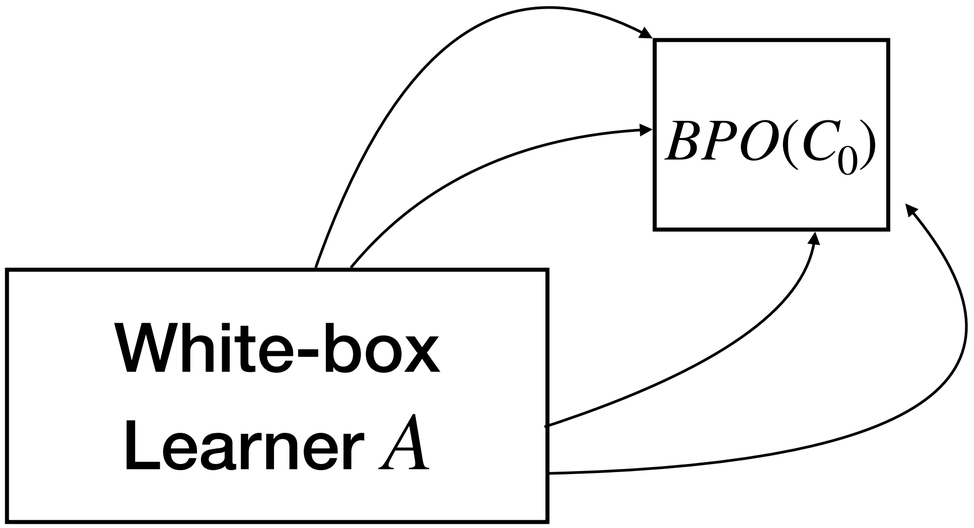

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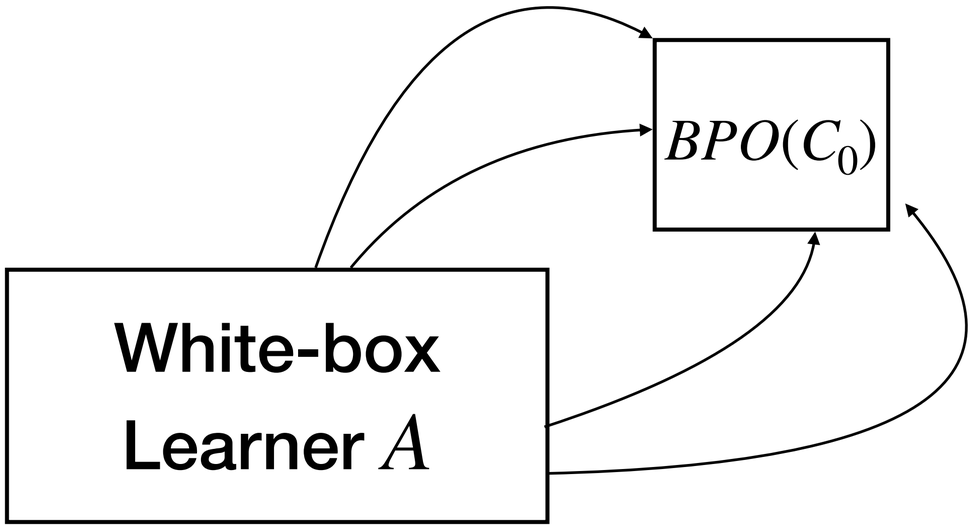

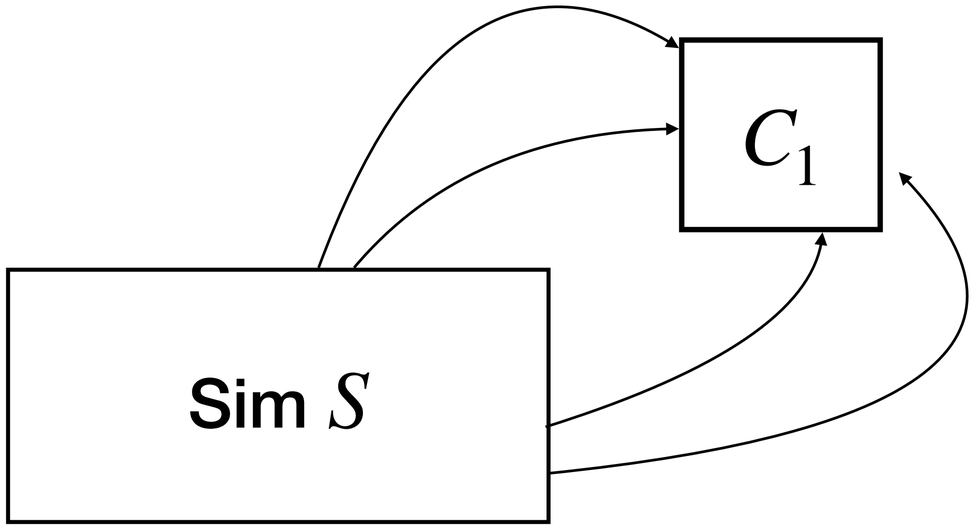

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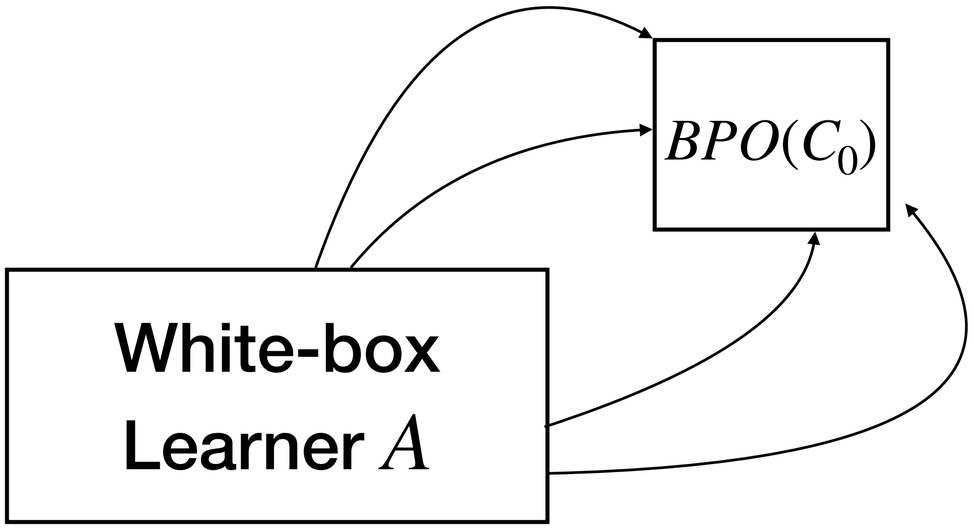

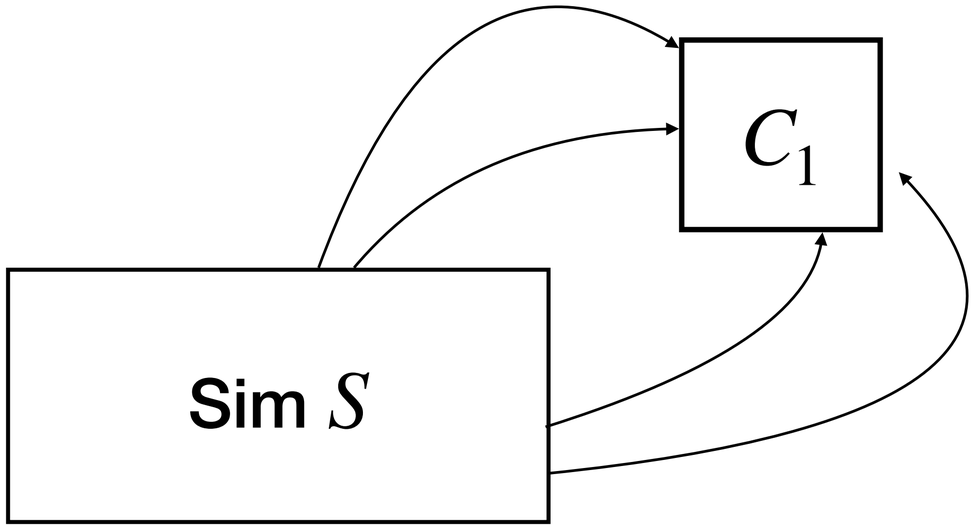

For all white-box learners A, there exists a simulator S such that

$$|\Pr[A(BPO(C_0)) = 1] - \Pr[S(C_1) = 1]| \leq \mathsf{negl}(|C|)$$

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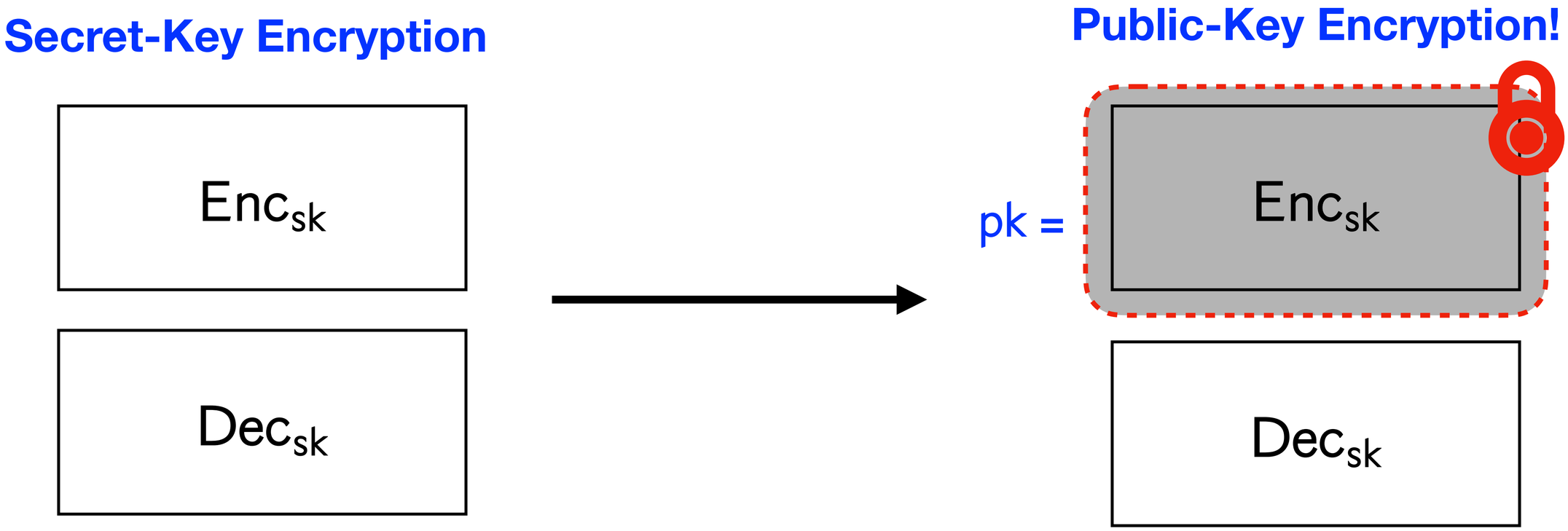
Corollary. If a circuit family has VBB obfuscation, then iO is a VBB obfuscation for this family.

Ok… But what can you do with iO?

iO Gymnastics

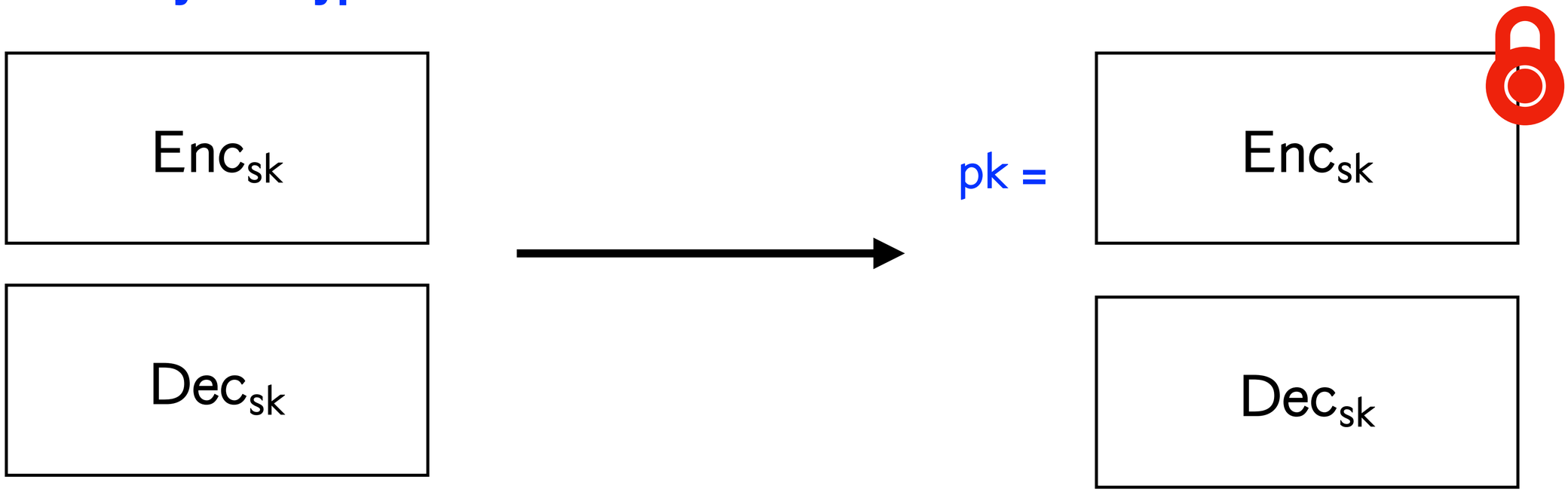
Recall: PKE from SKE!



OWF + VBB gives public key encryption!!

Recall: PKE from SKE!

Secret-Key Encryption



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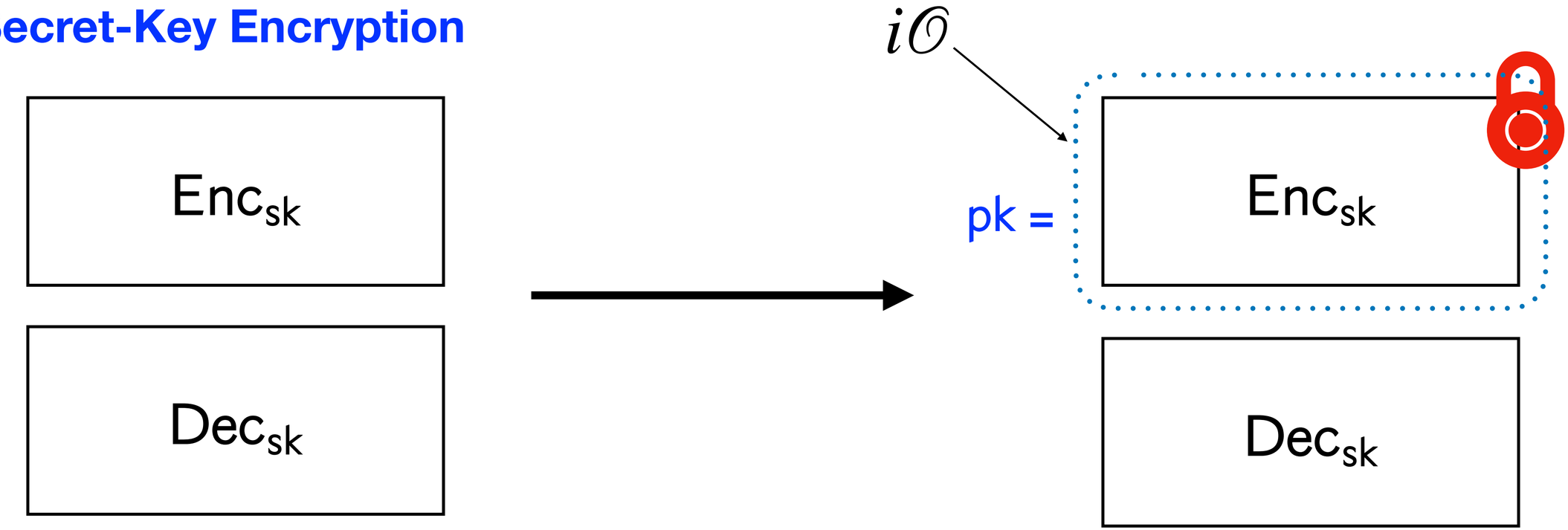

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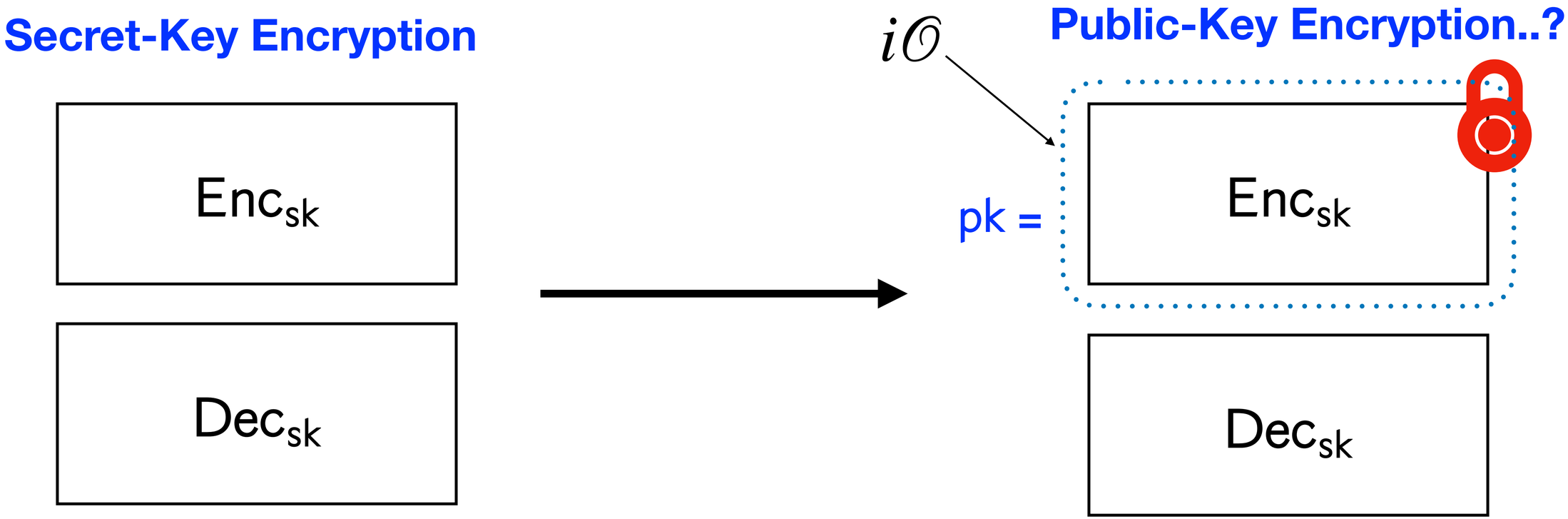

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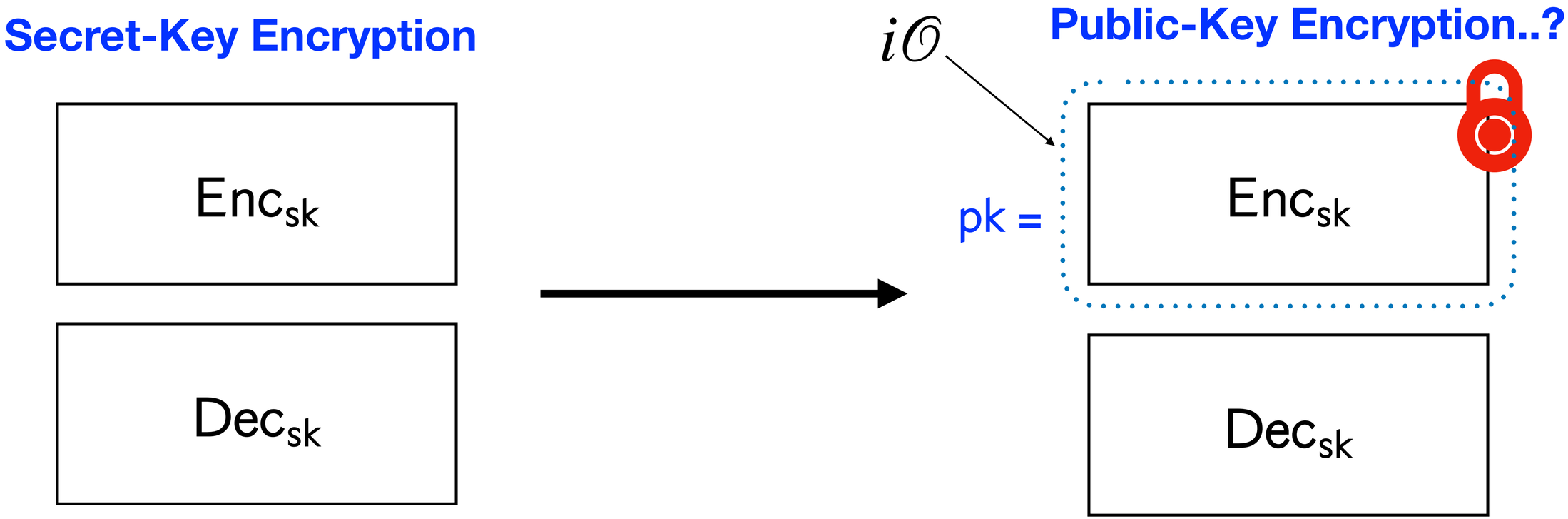

But… iO doesn't really gives us much to work with in this picture…

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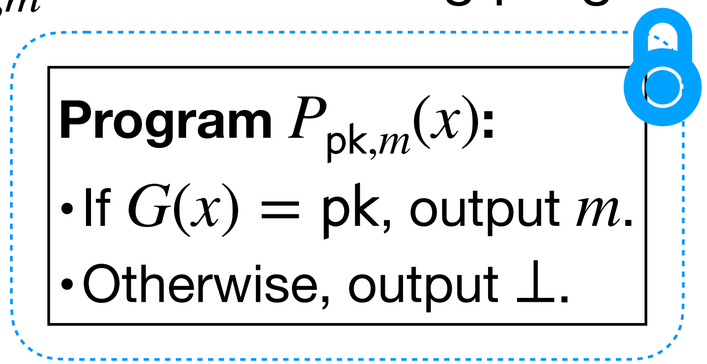

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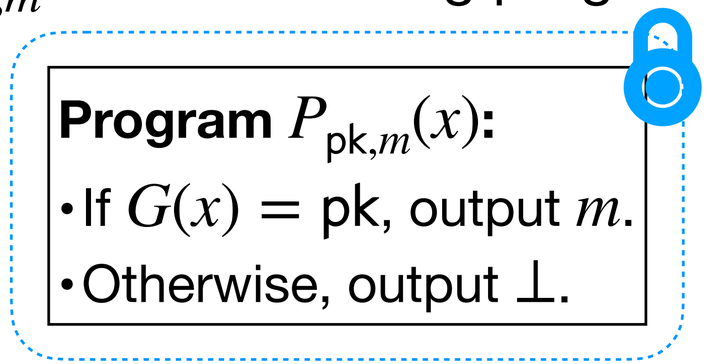

- Dec(sk, c): Interpret c as a program, run it on input $\mathsf{sk} = s$, and output the result.

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Proof: Want to show that $(\mathsf{pk}, Enc_{\mathsf{pk}}(m_0)) \approx_c (\mathsf{pk}, Enc_{\mathsf{pk}}(m_1))$.

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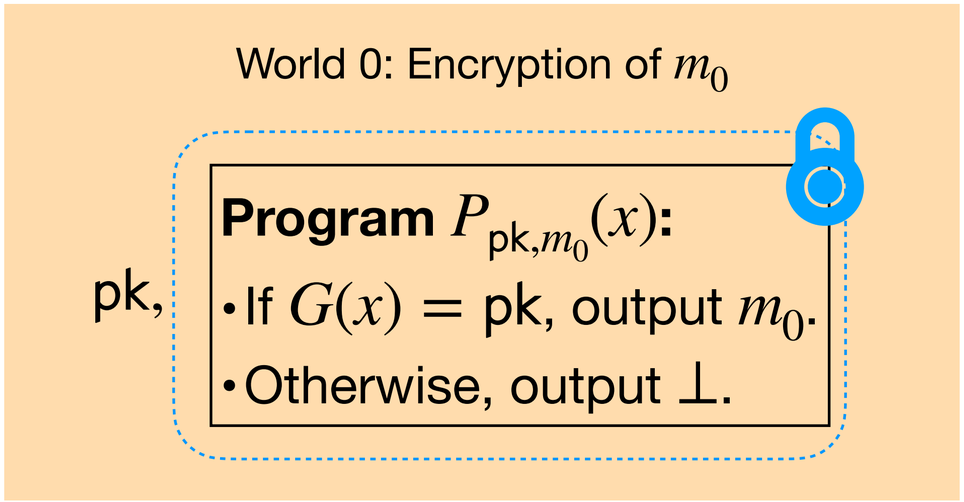

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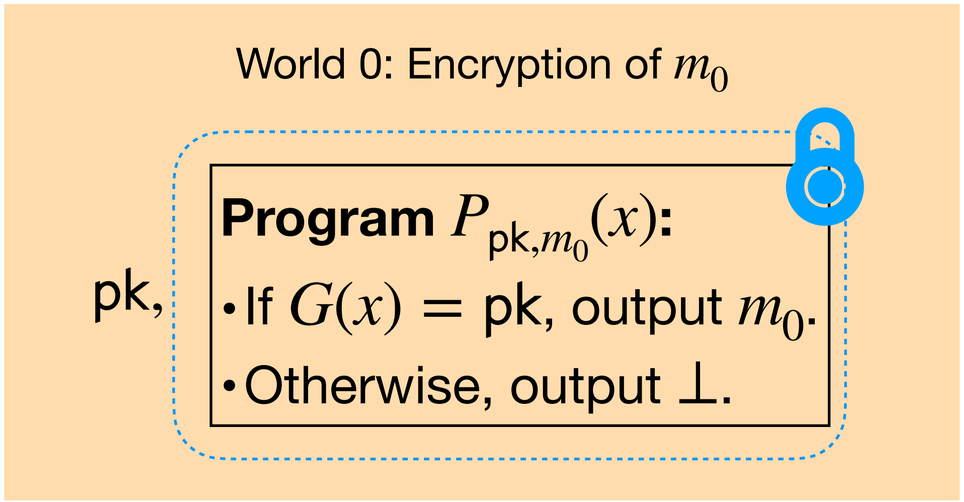

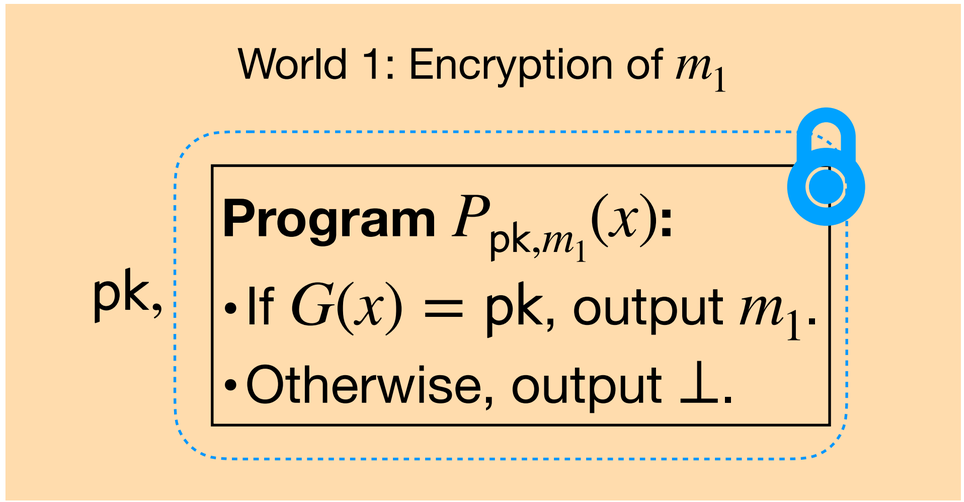

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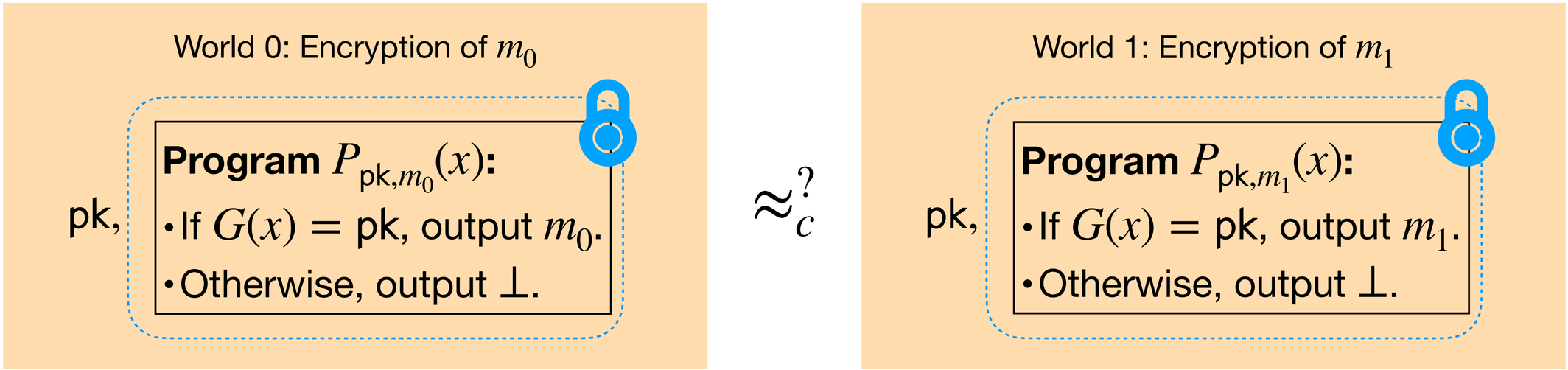

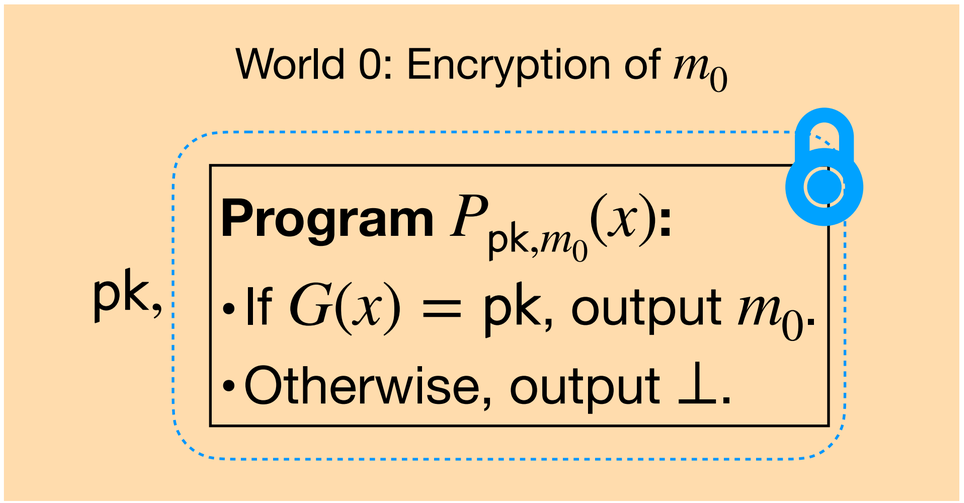
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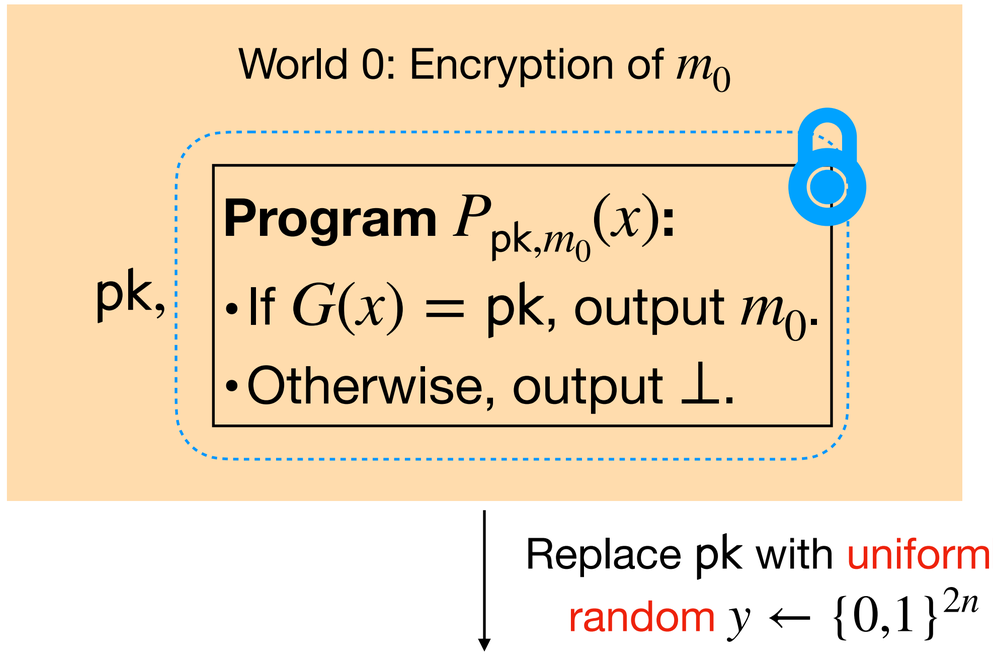

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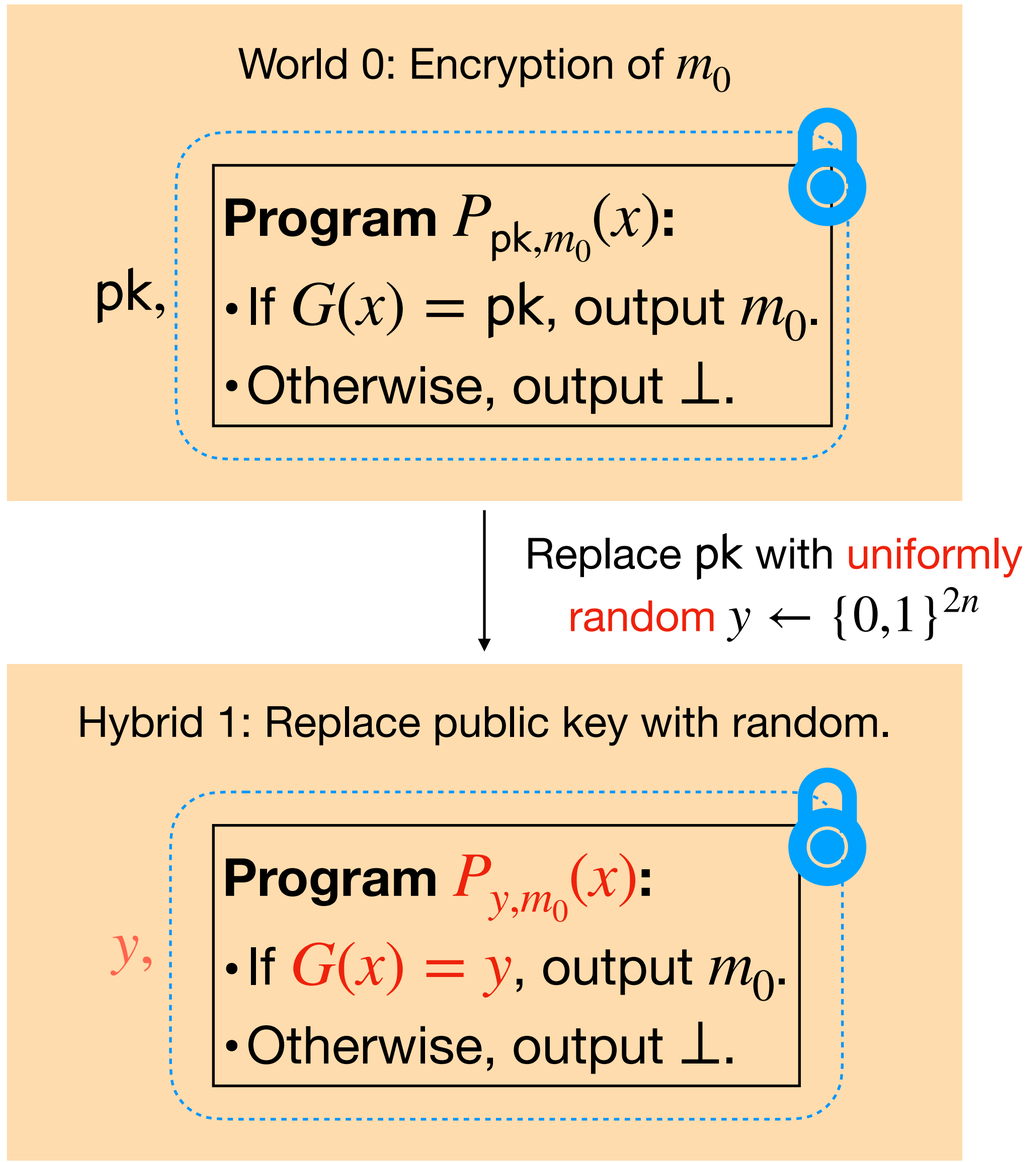
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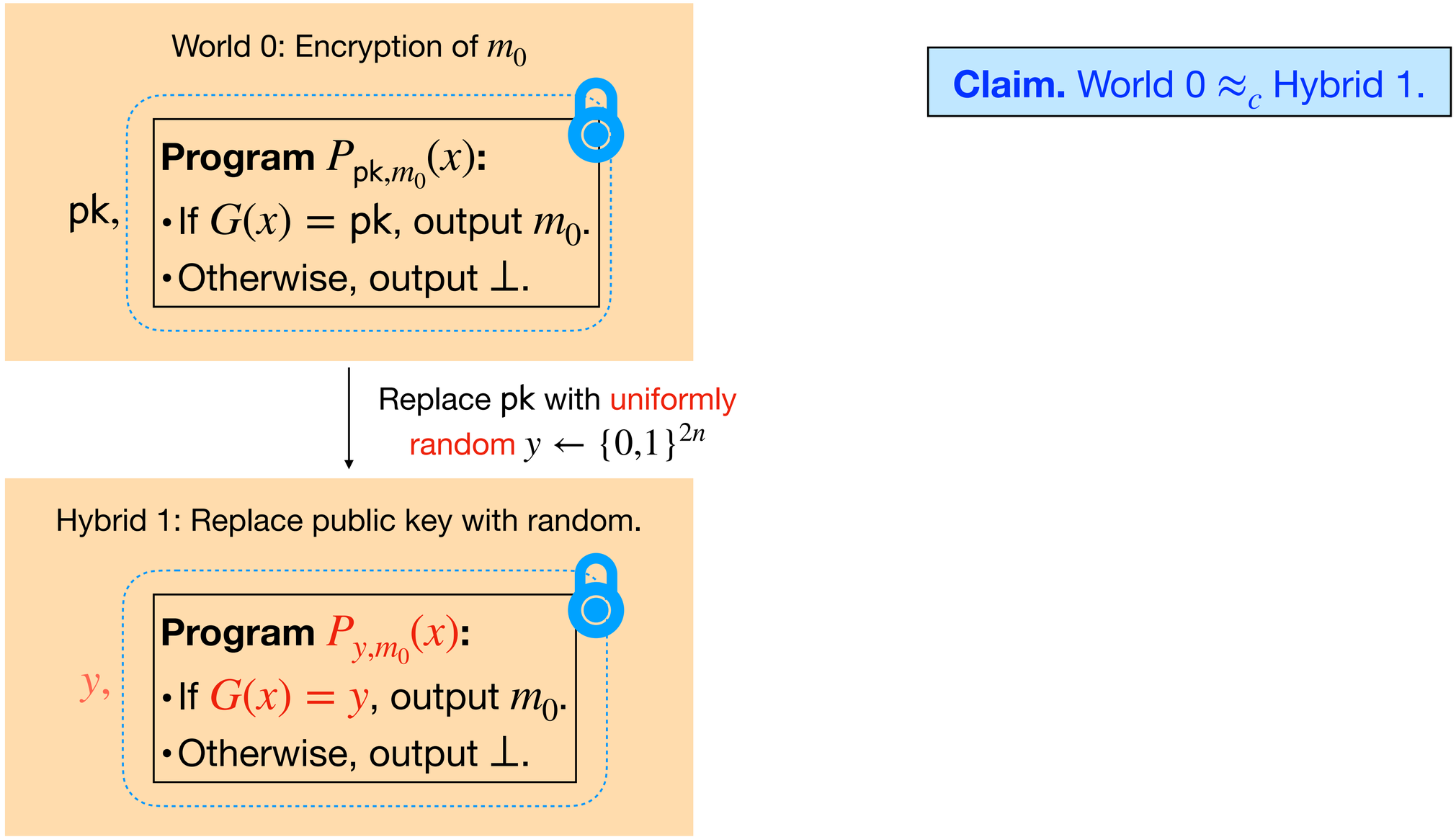

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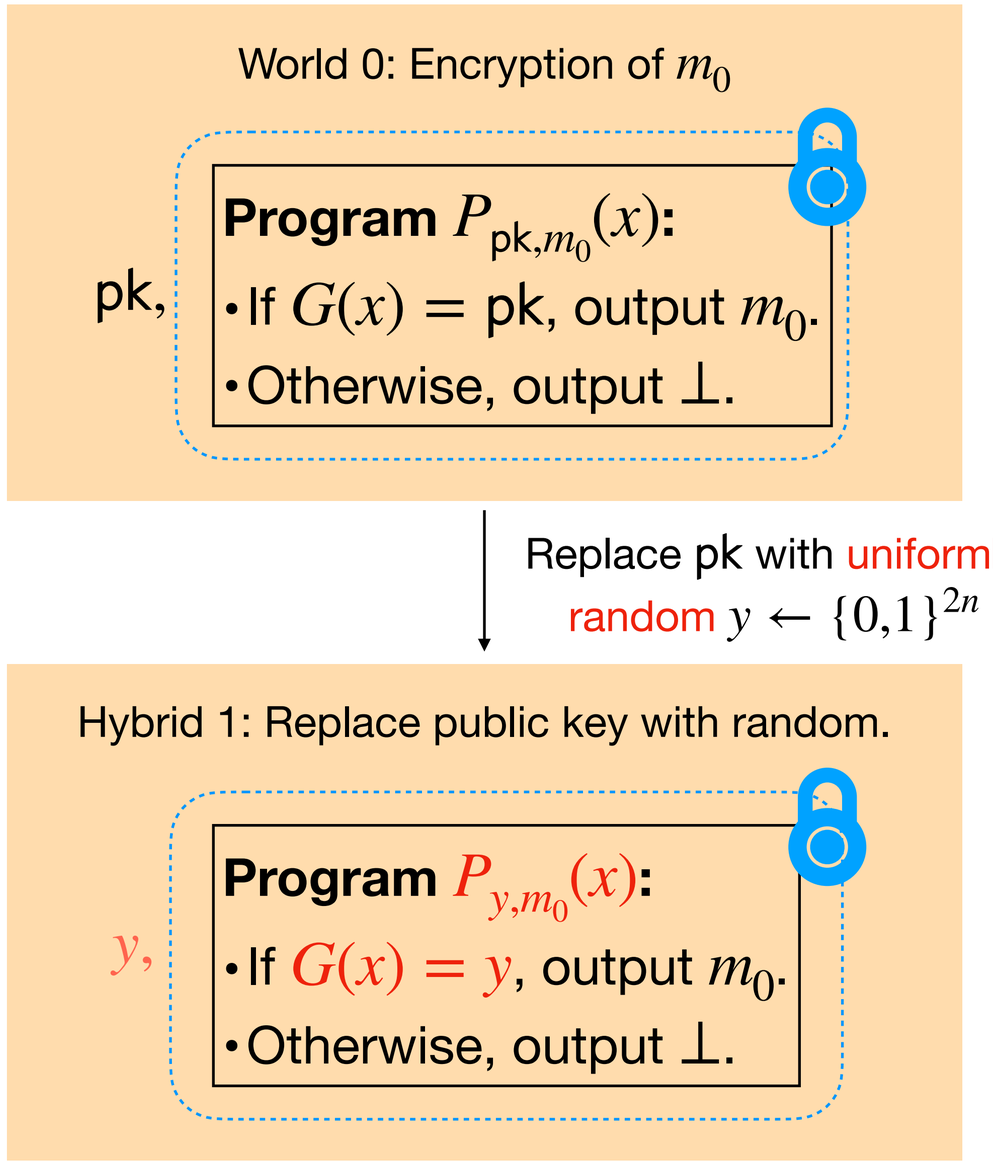
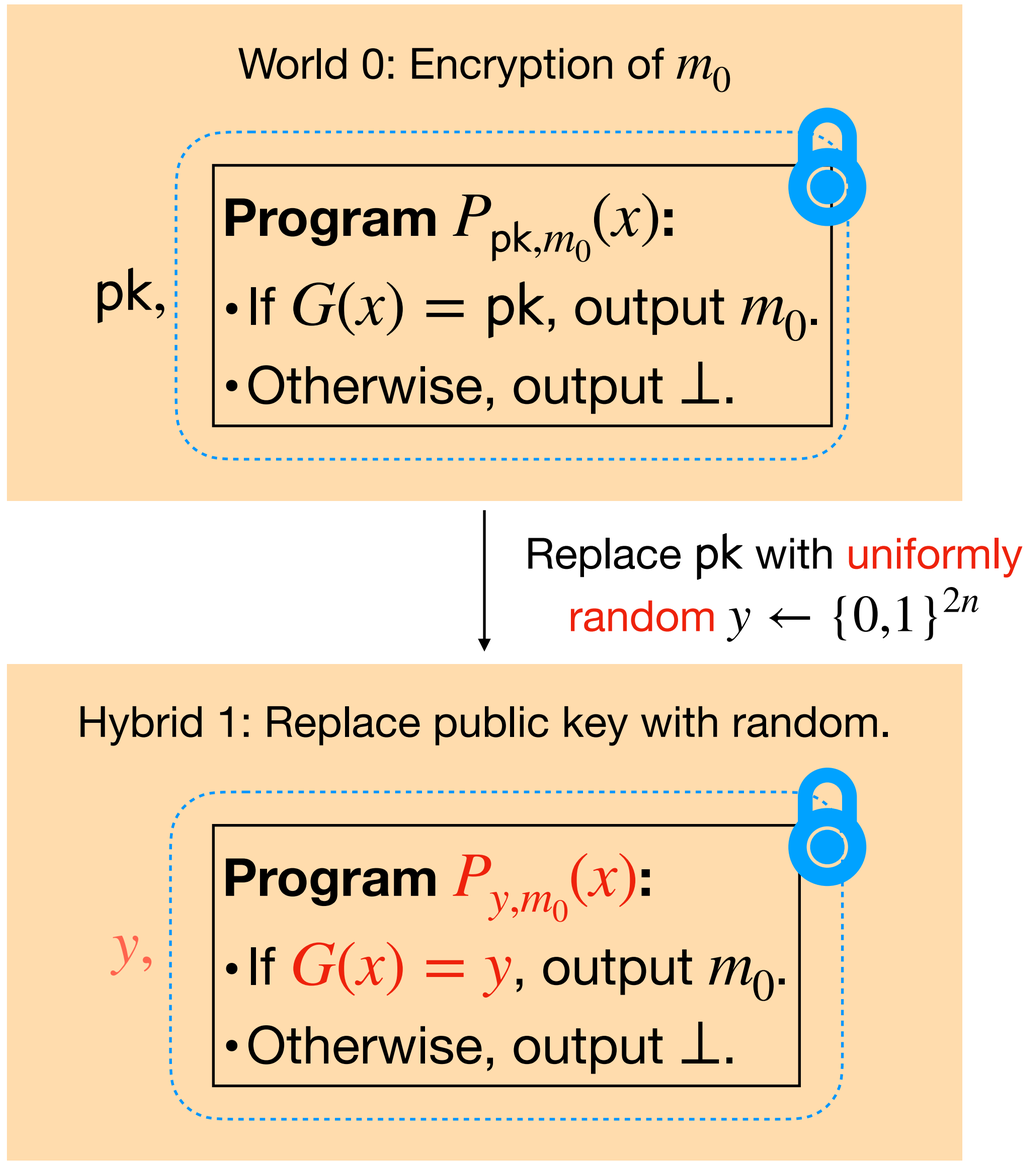


Claim. World 0 $\approx_c$ Hybrid 1.

If an adversary can distinguish these hybrids, then he breaks PRG security!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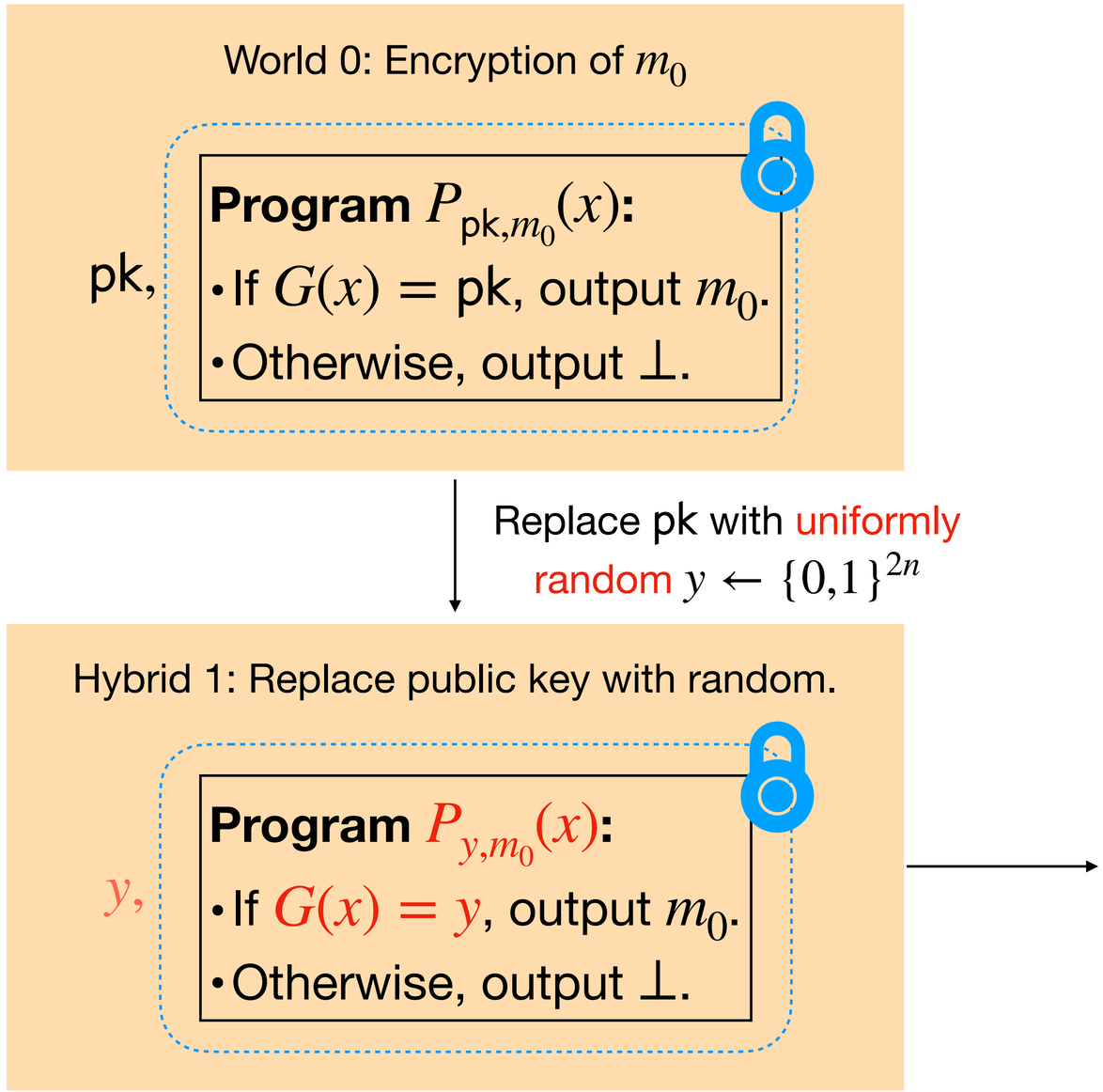
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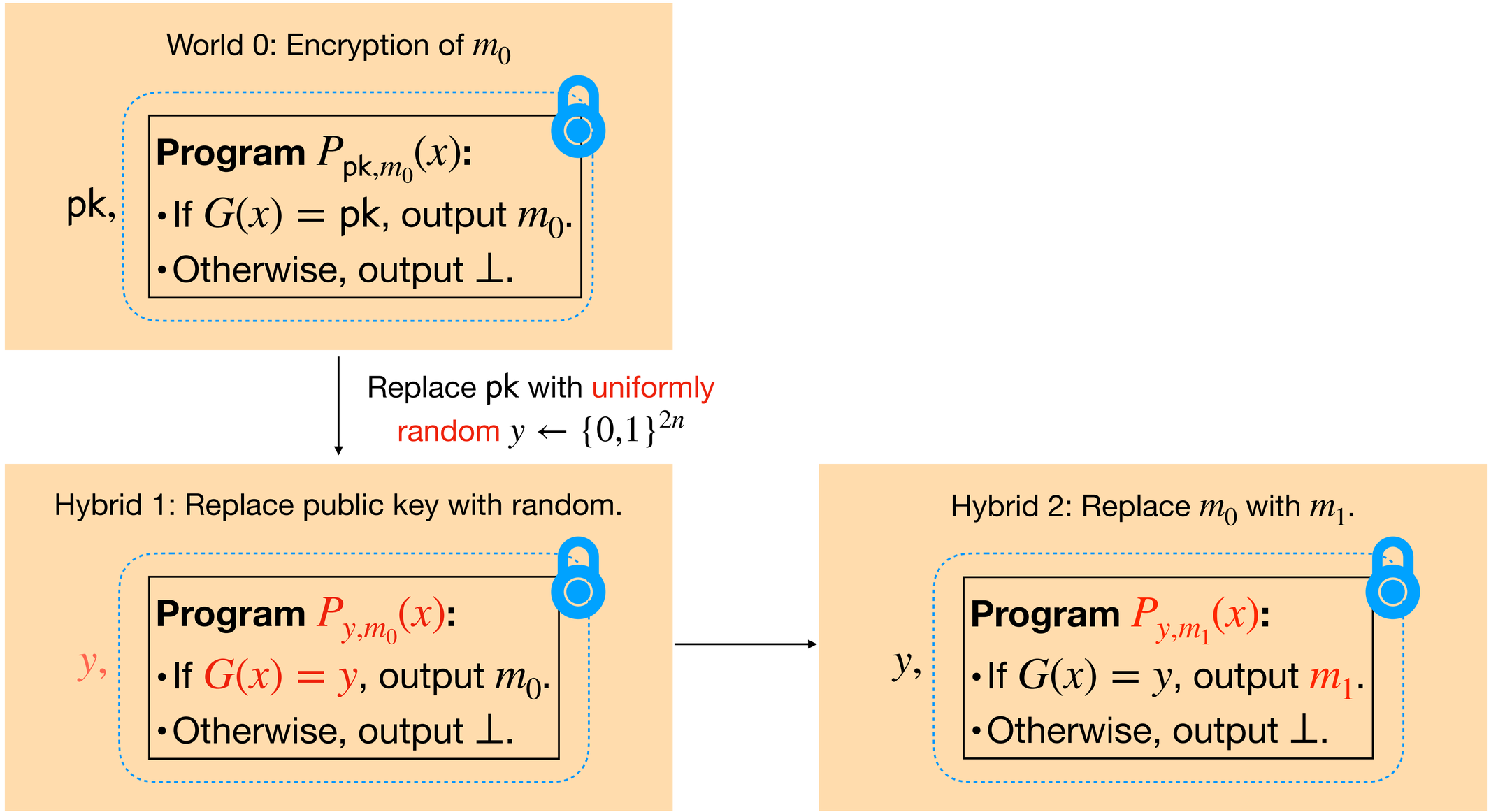
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Claim. Hybrid 1 $\approx_c$ Hybrid 2.

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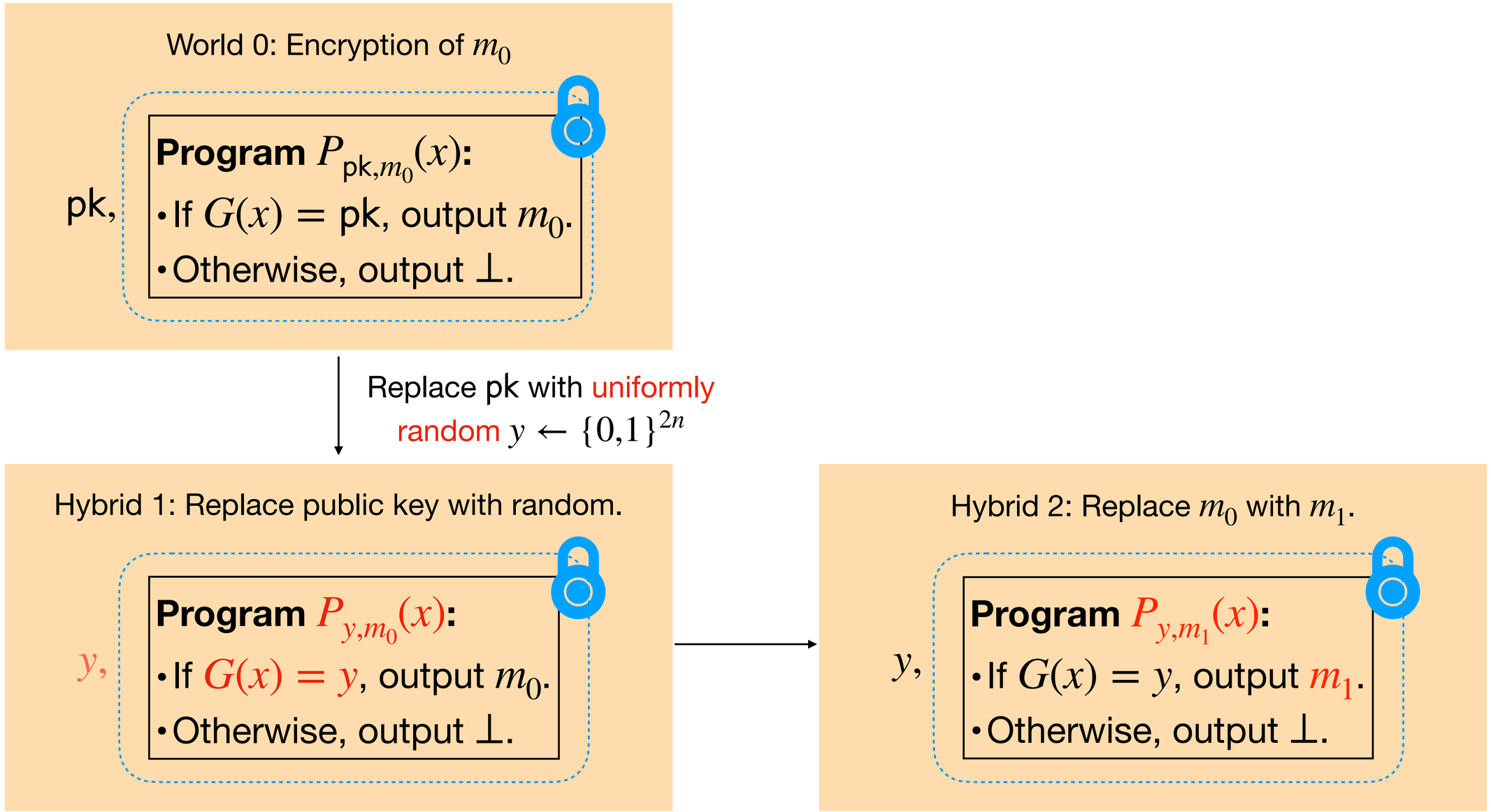
By $i\mathcal{O}$ security, $\widehat{P_{y,m_0}} \approx \widehat{P_{y,m_1}}$.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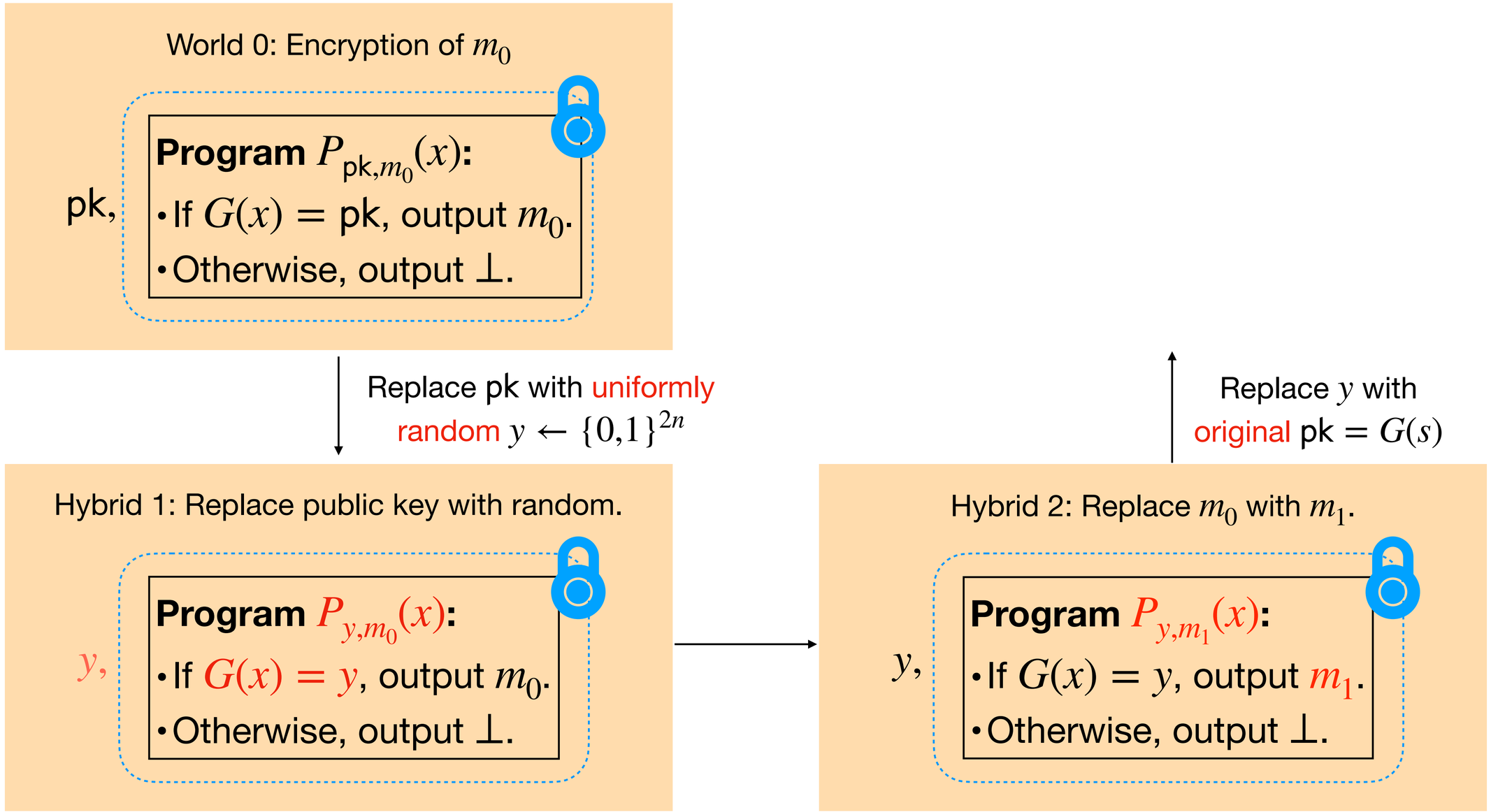
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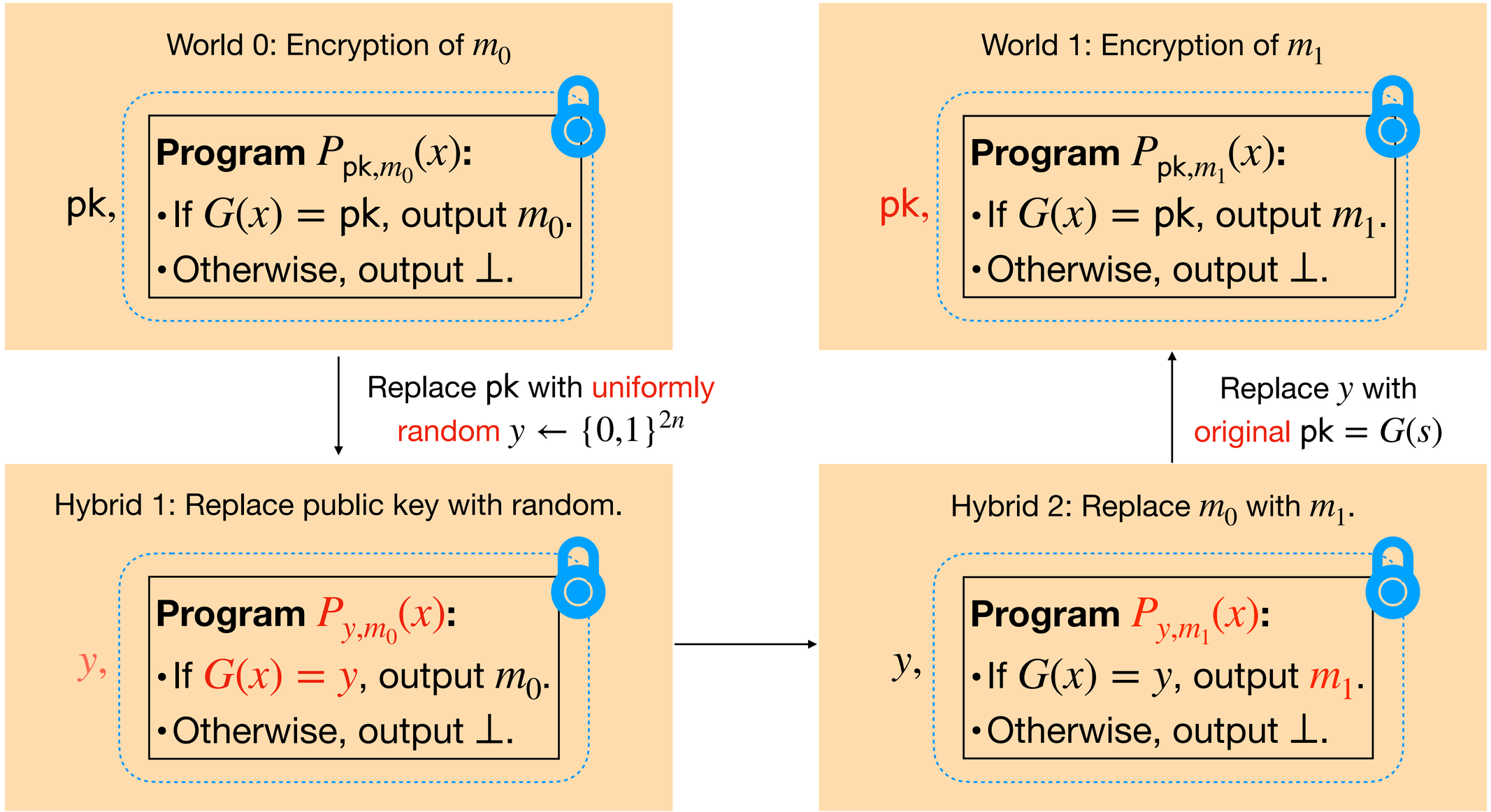
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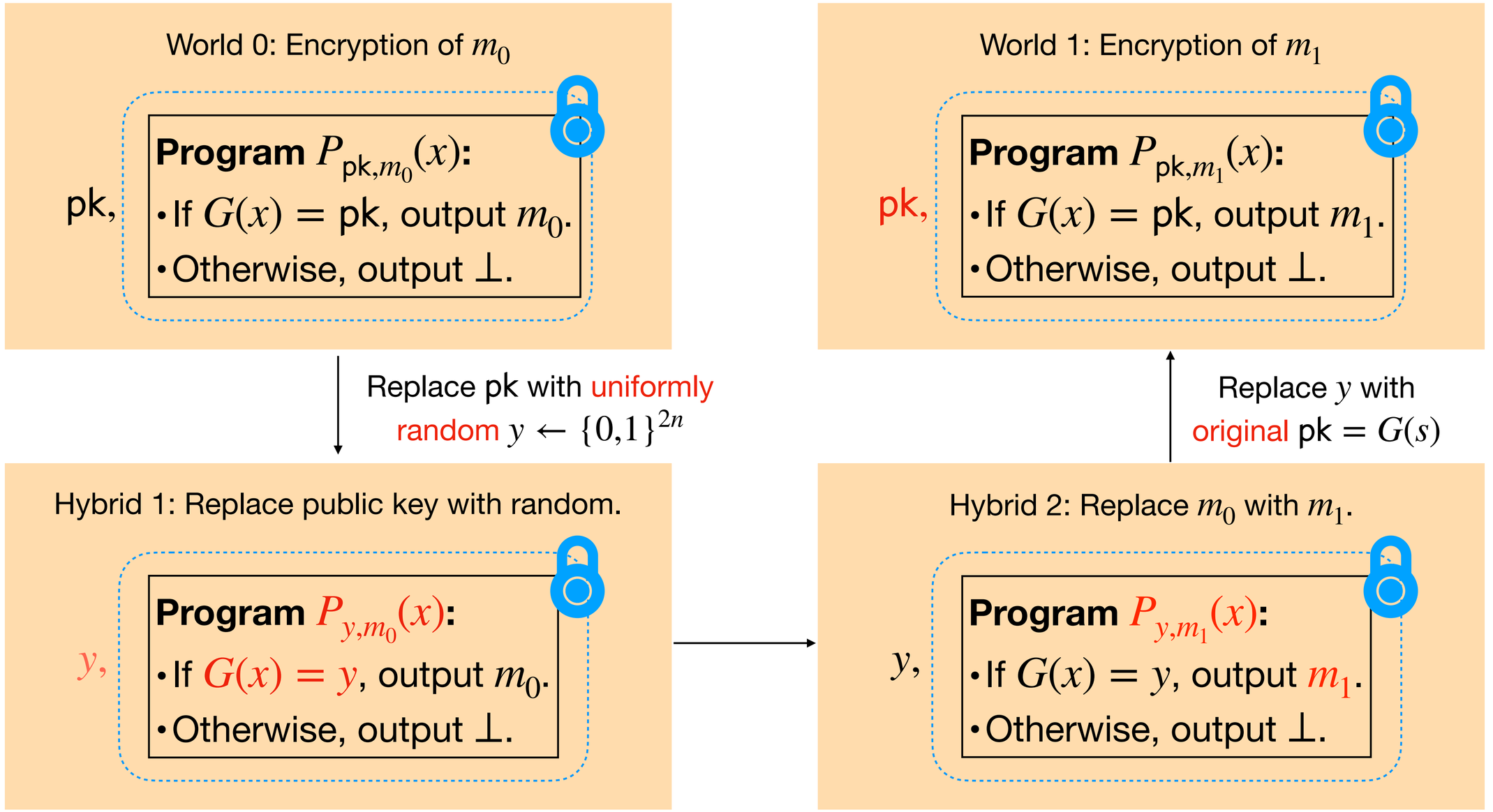
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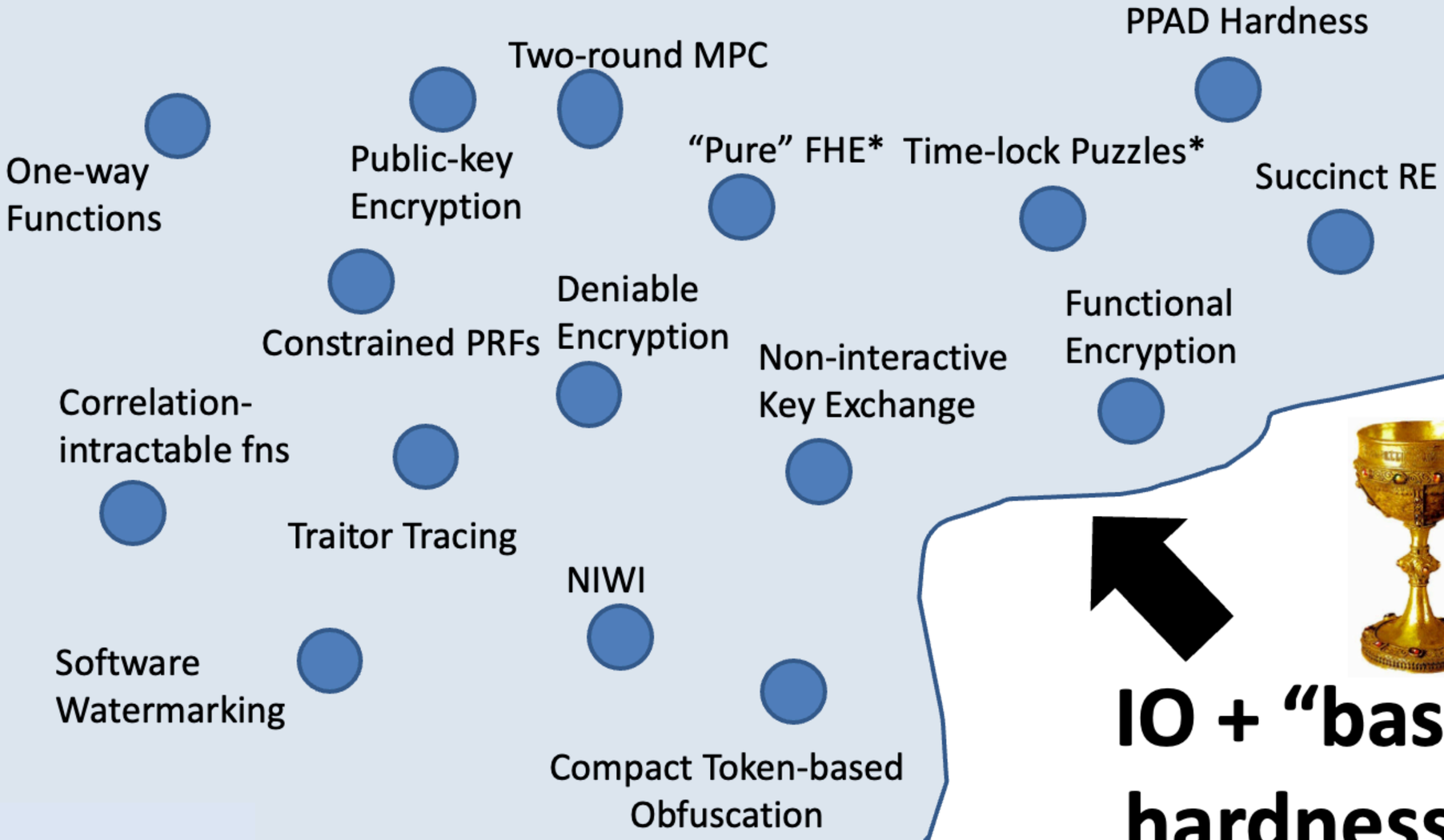
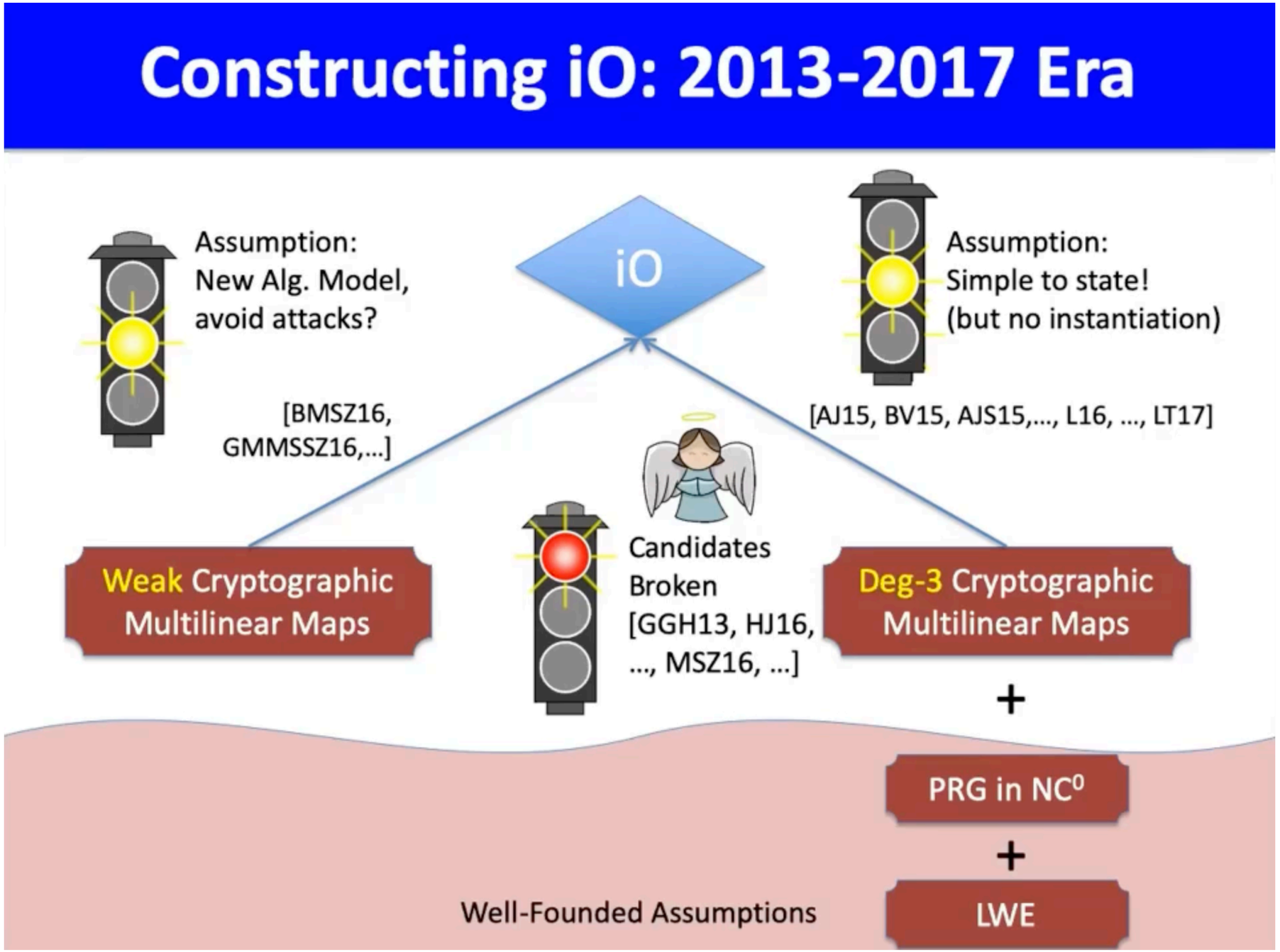
"CRYPTO-COMPLETE":

IO + Basic Hardness + Hard Work ⟹ Nearly all crypto.



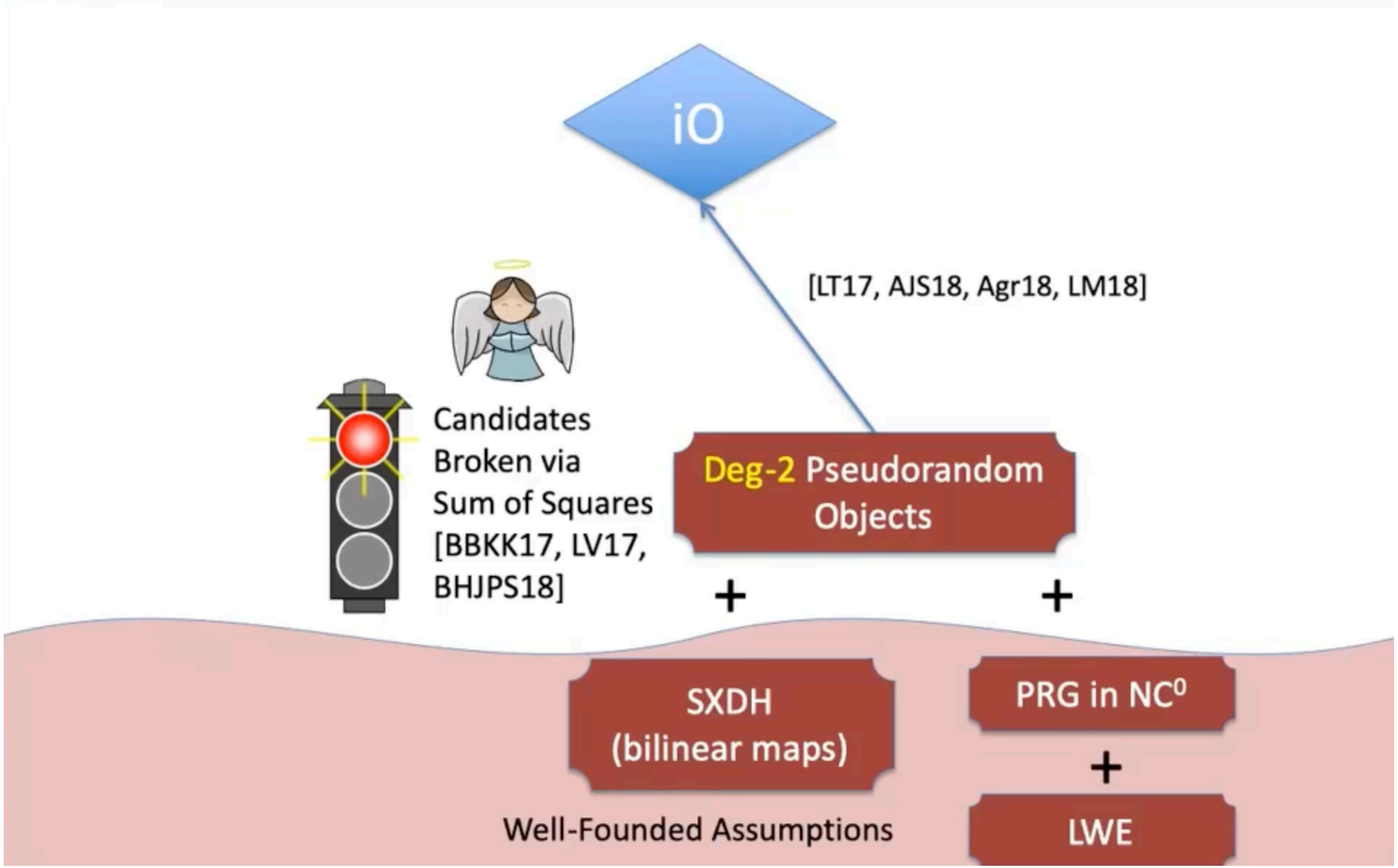
What is the state of iO constructions?

Constructing iO: 2013-2017 Era



From Amit Sahai's talk

Constructing iO: 2017-2018 Era



From Amit Sahai's talk

Indistinguishability Obfuscation from Well-Founded Assumptions

Aayush Jain* Huijia Lin† Amit Sahai‡

November 12, 2020

Abstract

Indistinguishability obfuscation, introduced by [Barak et. al. Crypto'2001], aims to compile programs into unintelligible ones while preserving functionality. It is a fascinating and powerful object that has been shown to enable a host of new cryptographic goals and beyond. However, constructions of indistinguishability obfuscation have remained elusive, with all other proposals relying on heuristics or newly conjectured hardness assumptions.

In this work, we show how to construct indistinguishability obfuscation from subexponential hardness of four well-founded assumptions. We prove:

Indistinguishability Obfuscation from Well-Founded Assumptions

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What next?

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Vinod’s Bounty

The construction of Jain, Lin and Sahai is not post-quantum secure.

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Can we get iO from just LWE?

Vinod's Bounty

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Witness Encryption and Null-IO from Evasive LWE

Vinod Vaikuntanathan*
MIT

Hoeteck Wee†
NTT Research

Daniel Wichs‡
Northeastern U. and NTT Research

August 31, 2022

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