MIT 6.875

Foundations of Cryptography
Lecture 23

Security against Malicious (Active) Adversaries

Secure Two-Party Comp: New Def

(possibly randomized) $F(x, y; r) = (F_A(x, y; r), F_B(x, y; r))$

Input: $\boldsymbol{x}$ **Input:** $\boldsymbol{y}$

Alice Bob

There exists a PPT simulator SIM_A such that for any x and y:

$$\big(SIM_A\big(x, F_A(x, y)\big), F(x, y)\big) \cong \big(View_A(x, y), F(x, y)\big)$$

i.e. the joint distribution of the view and the output is correct

Issues to Handle

1. Input (In)dependence: A malicious party could choose her input to depend on Bob's, something she cannot do in the ideal world.

Example (on the board): $F\big((a,b),x\big) = (\bot, ax+b)$

2. Randomness: A malicious party could choose her "random string" in the protocol the way she wants, something she cannot do in the ideal world.

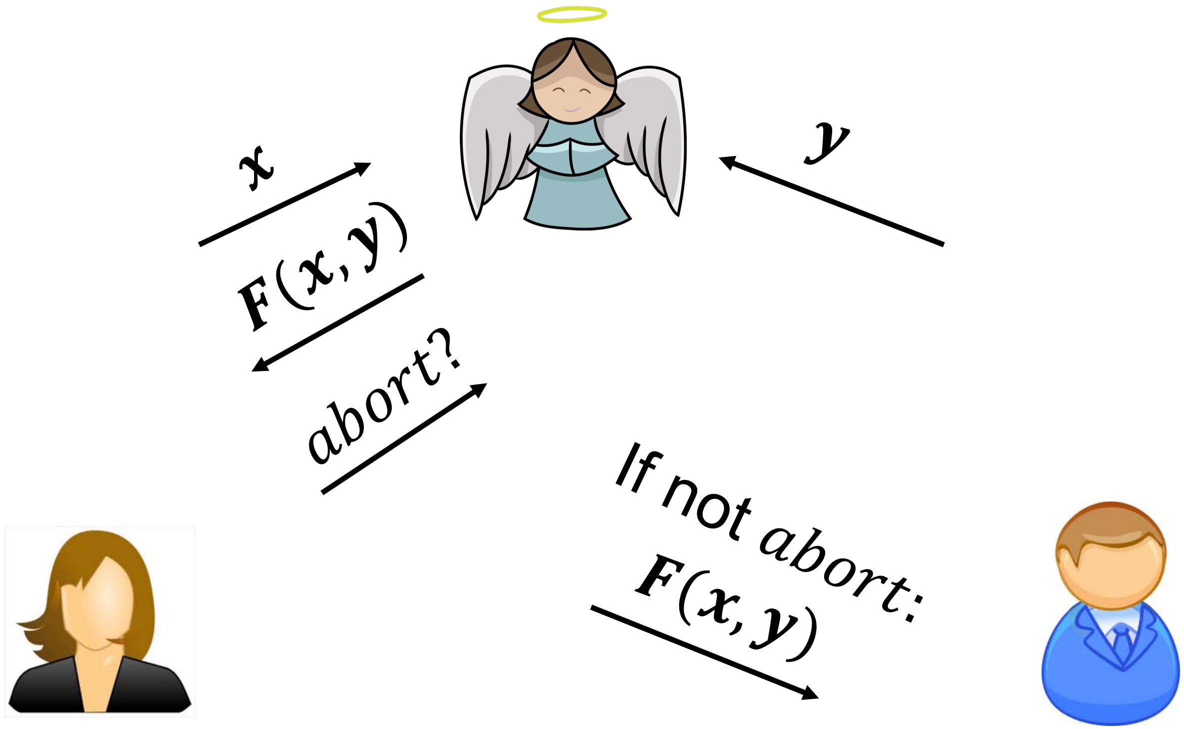
Example (on the board): our OT *protocol*

Cleve's theorem: this is unavoidable

3. (Un)fairness: A malicious party could block the honest party from learning the output, while learning it herself.

4. Deviate from Protocol Instructions.

New (Less) Ideal Model



The “GMW Compiler”

***Theorem* [Goldreich-Micali-Wigderson’87]**: Assuming one-way functions exist, there is a general way to transform any semi-honest secure protocol computing a (possibly randomized) function F into a maliciously secure protocol for F.

Input Independence

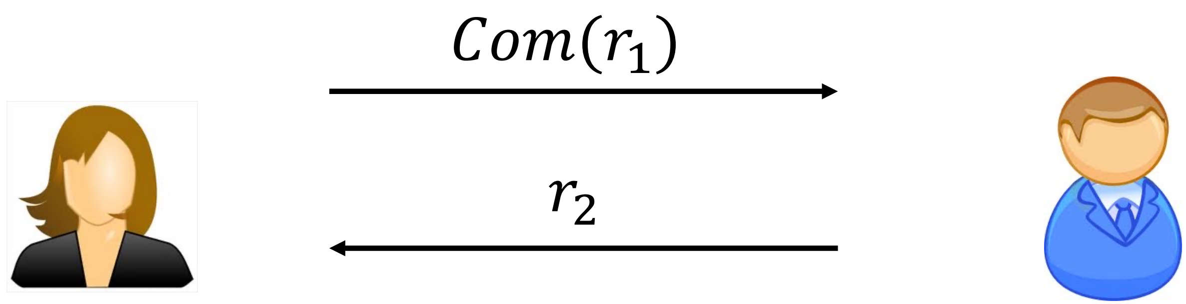
1. Input (In)dependence: A malicious party could choose her input to depend on Bob's, something she cannot do in the ideal world.

Solution: Each party commits to their input in sequence, and provides a **zero-knowledge proof of knowledge** of the underlying input.

Solution: Coin-Tossing Protocol

2. Randomness: A malicious party could choose her "random string" in the protocol the way she wants, something she cannot do in the ideal world.

__Def:__ Realize the functionality $F(1^n, 1^n) = (r, Com(r))$.



Output $r = r_1 \oplus r_2$

Output $(Com(r_1), r_2)$

Zero Knowledge Proofs

4. Deviate from Other Protocol Instructions.

<u>*Solution:*</u> Each message of each party is a *deterministic* function of their input, their random coins and messages from party B.

When party A sends a message $m = m(x_A, r_A, \overline{msg_B})$, they also prove in zero-knowledge that they did so correctly. That is, they prove in ZK the following NP statement:

$$(m, \overline{msg_B}, XCom, RCom):\ \exists\, x_A, r_A \text{ s.t.}$$
$$m = m(x_A, r_A, \overline{msg_B})\ \wedge XCom = \mathrm{Com}(x_A) \wedge$$
$$RCom = \mathrm{Com}(r_A)$$

Optimizations

Optimization 1: Preprocessing OTs

Random OT tuple (or AND tuple, or Beaver tuple after D. Beaver): Alice has (α, γ_a) and Bob has (β, γ_b) which are random s.t. $\boldsymbol{\gamma_a \oplus \gamma_b = \alpha\beta}$.

Theorem: Given O(1) many *random* OT tuples, we can do OT with information-theoretic security, exchanging O(1) bits.

Optimization 2: OT Extension

Theorem
[Beaver'96, Ishai-Kushilevitz-Nissim-Pinkas'03]:

Given O(λ) many *random* OT tuples, we can generate n OT tuples exchanging O(n) bits --- as opposed to the trivial O($n\lambda$) bits --- and using only symmetric-key crypto.

Complexity of the 2-party solution

Number of OT protocol invocations $= 2 \ * \ \#AND$ gates

Can be made into O(#inputs · λ): Yao's garbled circuits

Number of rounds = AND-depth of the circuit

Can be made into O(1) rounds: Yao's garbled circuits

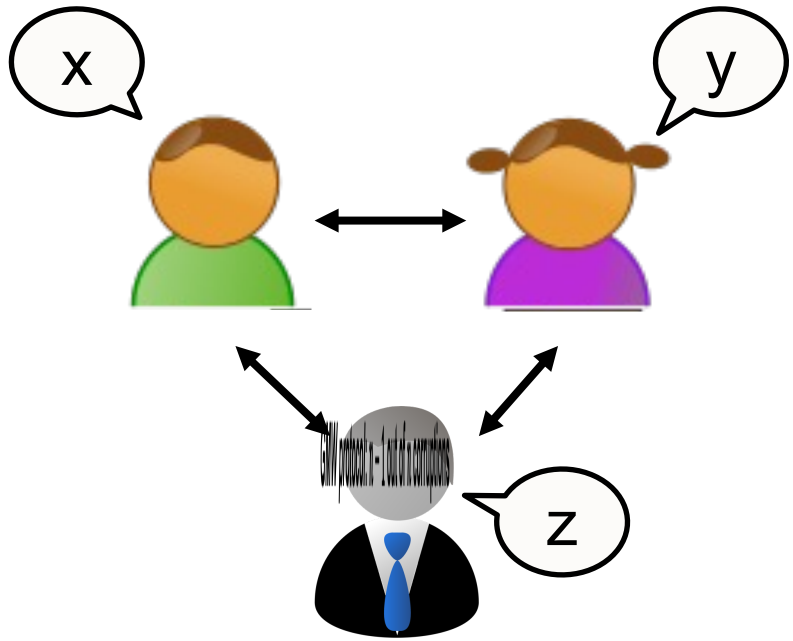
Communication in bits =

$$O(\#AND \cdot \lambda + \#outputs)$$

Can be made into O(λ #inputs) using FHE: but FHE is computationally more expensive concretely.

Secure Multi Party Computation w/ Information-theoretic security

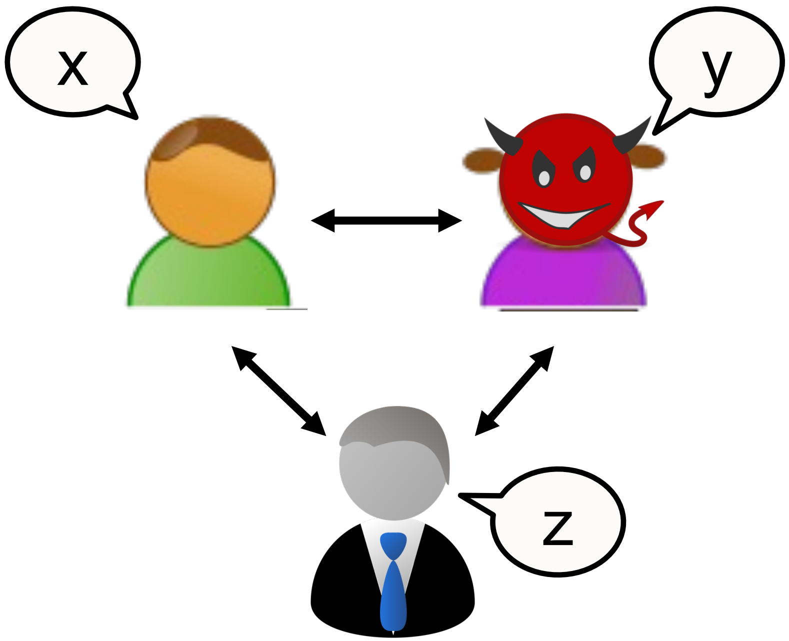
Secure *Multi*-party Computation



***GMW protocol*: $n - 1$ out of n corruptions**

The Goldreich-Micali-Wigderson Protocol for n parties with $< n$ corruptions, using Oblivious Transfer
(which can only be *computationally* secure)

Secure *Multi*-party Computation



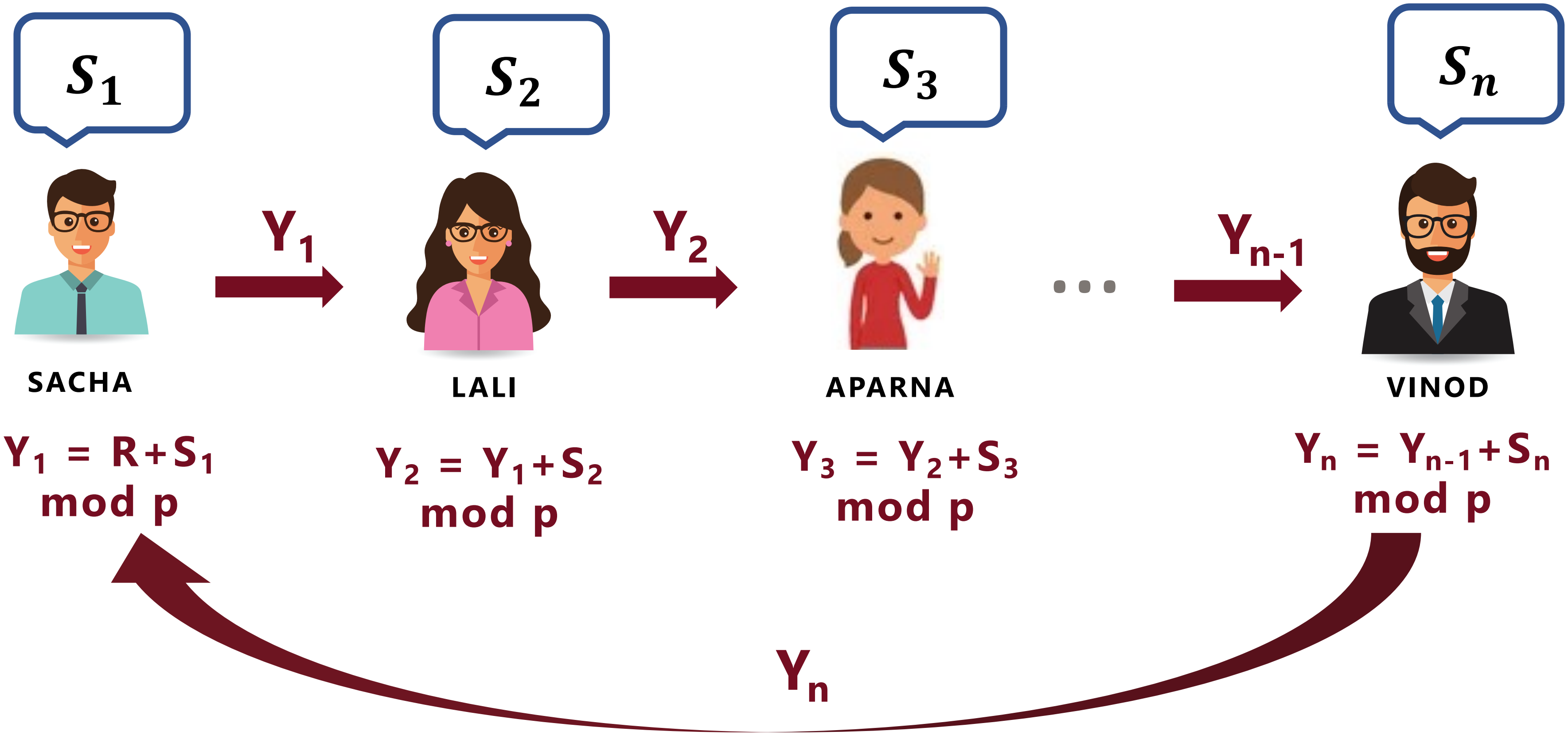
***TODAY*: HONEST MAJORITY**

Information-theoretically Secure Protocols for n parties with $< n/2$ corruptions

[BenOr-Goldwasser-Wigderson'88, Chaum-Crepeau-Damgard'88, BenOr-Rabin'89]

AN EXAMPLE

COMPUTING THE AVERAGE SALARY IN THIS ROOM



$$Y_n - R = \sum_{i=1}^{n} S_i \text{ mod p} = \sum_{i=1}^{n} S_i$$

(if p large enough)

Is this secure?

IT-Secure MPC with Honest Majority

***Theorem* [BenOr-Goldwasser-Wigderson'88, Chaum-Crepeau-Damgard'88]**:

Any n-party computation problem can be solved with information-theoretic security as long $< \frac{n}{2}$ parties collude.

***Key Tool:* Shamir's Secret Sharing**

Key Tool: Secret-Sharing

- Any **"authorized"** subset of players **can recover** b.
- No other subset of players **has any info** about b.

- Threshold (or t-out-of-n) SS [Shamir'79, Blakley'79]:

  "authorized" subset = has size $\geq$ t.

secret b $\in Z_p$

n-out-of-n Secret Sharing

Dealer

P_1 P_2 P_3 P_4 … P_n

share s_1: random

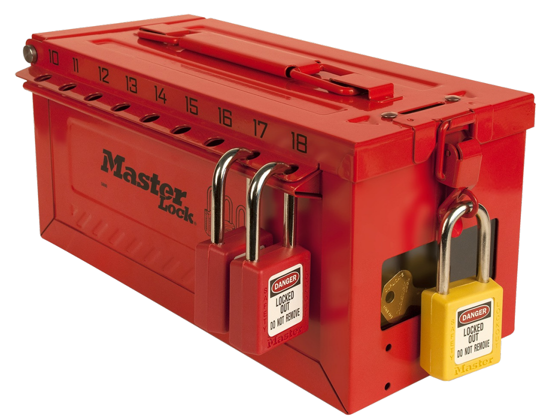
share s_2: random

share s_3: random

share s_4: random

…

share $s_n = b - (s_1 + s_2 + \cdots + s_{n-1})$ mod p



secret b $\in Z_p$

1-out-of-n Secret Sharing

Dealer

P_1 P_2 P_3 P_4 … P_n

share s_1 = b

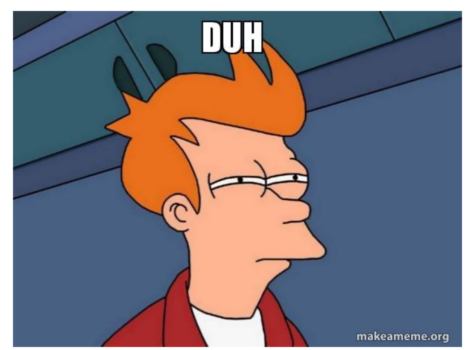
share s_2 = b

share s_3 = b

share s_4 = b

…

share $s_n = b$



secret $b \in Z_p$

2-out-of-n Secret Sharing?

Dealer P_1 P_2 P_3 P_4 … P_n

Here is a solution.

Repeat for every two-person subset $\{P_i, P_j\}$:

- Generate a 2-out-of-2 secret sharing (s_i, s_j) of b.
- Give s_i to P_i and s_j to P_j

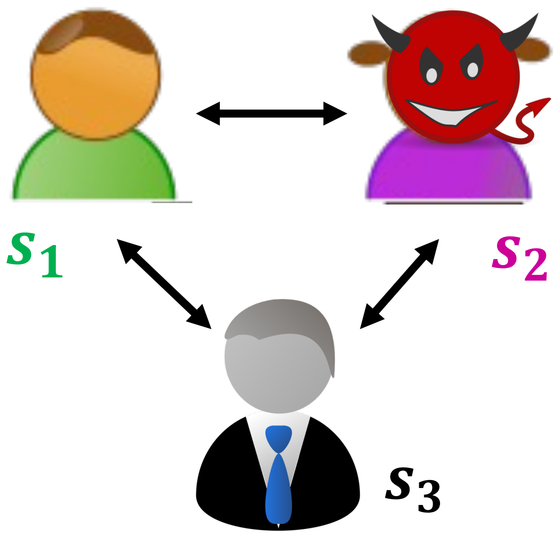
What is the size of shares each party gets?

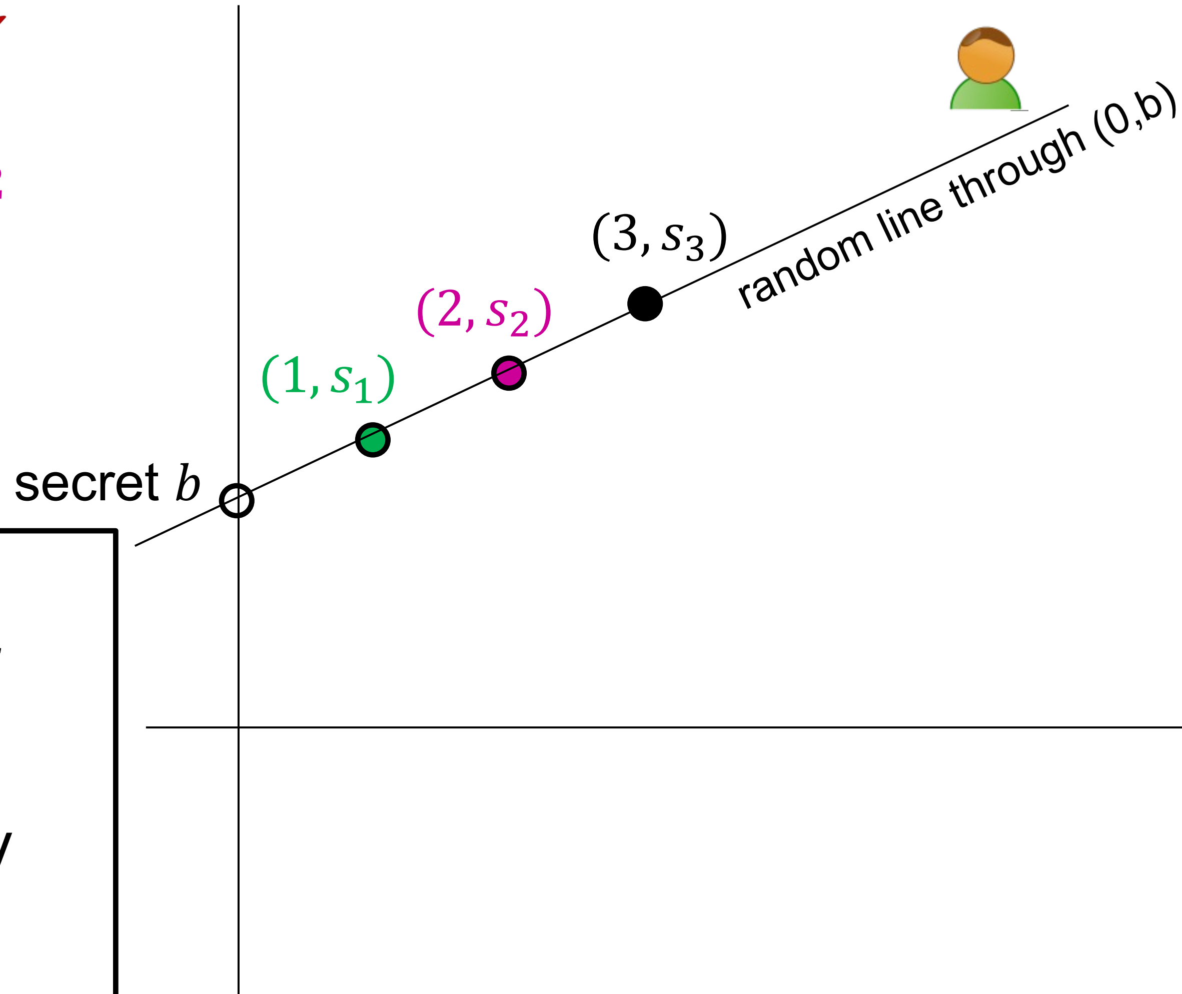
How does this scale to t-out-of-n?

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

Shamir's 2-out-of-n Secret Sharing





Each share s_i is truly random (independent of secret b)

Any two shares uniquely determine b.

Shamir's 2-out-of-n Secret Sharing

1. The dealer picks a uniformly random line **(mod p)** whose constant term is the secret b.

$f(x) = ax + b$ where a is uniformly random mod p

2. Compute the shares:

$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Correctness: can recover secret from any two shares.

Proof: Parties i and j, given shares $s_i = ai + b$ and $s_j = aj + b$ can solve for b $\left(= \frac{js_i - is_j}{j-i}\right)$.

Shamir's 2-out-of-n Secret Sharing

1. The dealer picks a uniformly random line **(mod p)** whose constant term is the secret b.

   $f(x) = ax + b$ where a is uniformly random mod p

2. Compute the shares:

   $$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Security: any single party has no information about the secret.

Proof: Party i's share $s_i = a * i + b$ is uniformly random, independent of b, as a is random and so is $a * i$.

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

1. The dealer picks a uniformly random degree-(t-1) polynomial **(mod p)** whose constant term is the secret b.

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

2. Compute the shares:

$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Correctness: can recover secret from any t shares.

Security: the distribution of any $t - 1$ shares is independent of the secret.

Note: need p to be larger than the number of parties n.

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Correctness: *via Vandermonde matrices*.

Let's look at shares of parties $P_1, P_2, \ldots, P_t$.

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \ldots \\ s_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & 2 & 2^2 & \ldots & 2^{t-1} \\ 1 & 3 & 3^2 & \ldots & 3^{t-1} \\ 1 & \ldots & \ldots & \ldots & \ldots \\ 1 & t & t^2 & \ldots & t^{t-1} \end{bmatrix} \begin{bmatrix} b \\ a_1 \\ a_2 \\ \ldots \\ a_{t-1} \end{bmatrix} \pmod{p}$$

*t-by-t Vandermonde matrix which is **invertible***

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Correctness: Alternatively, *Lagrange interpolation* gives an explicit formula that recovers b.

$$b = f(0) = \sum_{i=1}^{t} f(i) \left(\prod_{1 \le j \le t, j \ne i} \frac{-x_j}{x_i - x_j} \right)$$

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

Security:

Let's look at shares of parties $P_1, P_2, \ldots, P_{t-1}$.

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \ldots \\ s_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & 2 & 2^2 & \ldots & 2^{t-1} \\ 1 & 3 & 3^2 & \ldots & 3^{t-1} \\ 1 & \ldots & \ldots & \ldots & \ldots \\ 1 & t-1 & (t-1)^2 & \ldots & (t-1)^{t-1} \end{bmatrix} \begin{bmatrix} b \\ a_1 \\ a_2 \\ \ldots \\ a_{t-1} \end{bmatrix} \pmod p$$

$(t-1)$-by-t Vandermonde matrix

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

$$s_1 = f(1), s_2 = f(2), \dots, s_i = f(i), \dots, s_n = f(n)$$

Security: For every value of b there is a unique polynomial with constant term b and shares $s_1, s_2, \dots, s_{t-1}$.

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \dots \\ s_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2^2 & \dots & 2^{t-1} \\ 1 & 3 & 3^2 & \dots & 3^{t-1} \\ 1 & \dots & \dots & \dots & \dots \\ 1 & t-1 & (t-1)^2 & \dots & (t-1)^{t-1} \end{bmatrix} \begin{bmatrix} b \\ a_1 \\ a_2 \\ \dots \\ a_{t-1} \end{bmatrix} \pmod{p}$$

$(t-1)$-by-t Vandermonde matrix

Shamir's t-out-of-n Secret Sharing

Key Idea: Polynomials are Amazing!

$$f(x) = a_{t-1}x^{t-1} + \cdots + a_1 x + b$$

where a_i are uniformly random mod p

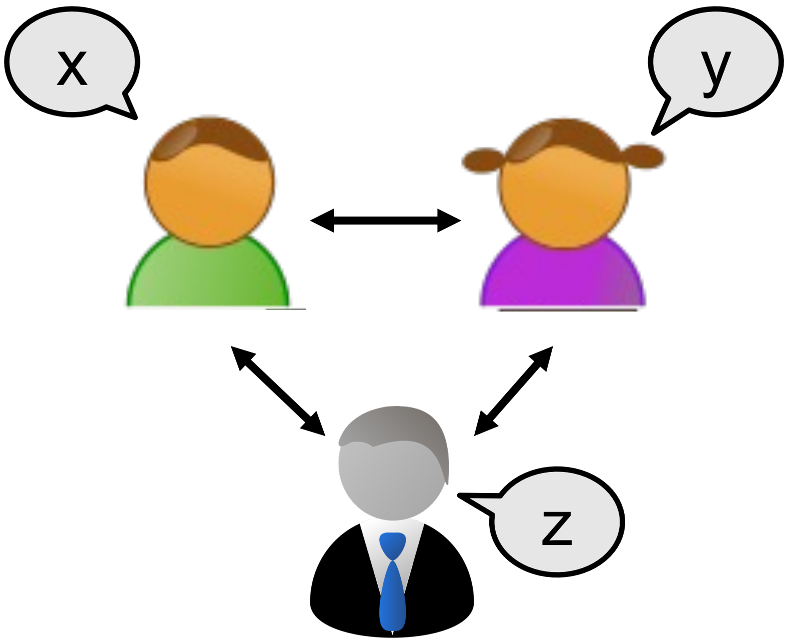
$$s_1 = f(1), s_2 = f(2), \ldots, s_i = f(i), \ldots, s_n = f(n)$$

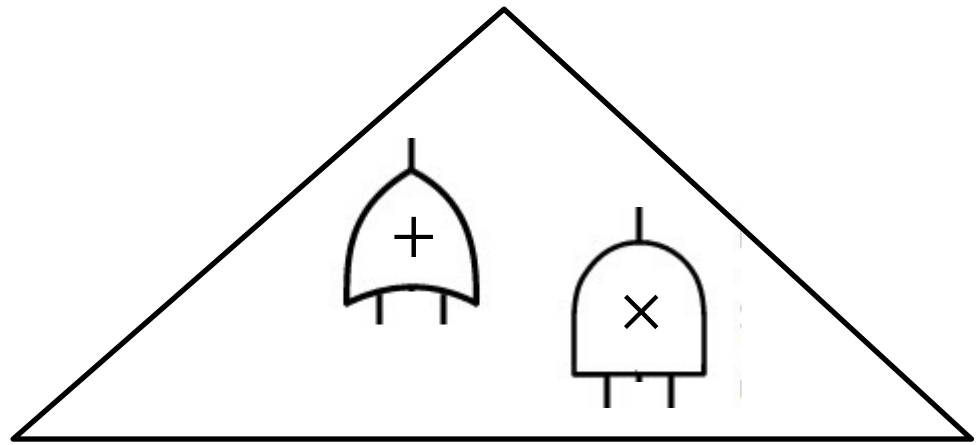
Security: For every value of b there is a unique polynomial with constant term b and shares $s_1, s_2, \ldots, s_{t-1}$.

Corollary: for every value of the secret b is equally likely given the shares $s_1, s_2, \ldots, s_{t-1}$. In other words, the secret b is perfectly hidden given $t-1$ shares.

Secure Multiparty Computation

[BenOr-Goldwasser-Wigderson'88, Chaum-Crepeau-Damgard'88, BenOr-Rabin'89]





Arithmetic circuit for F

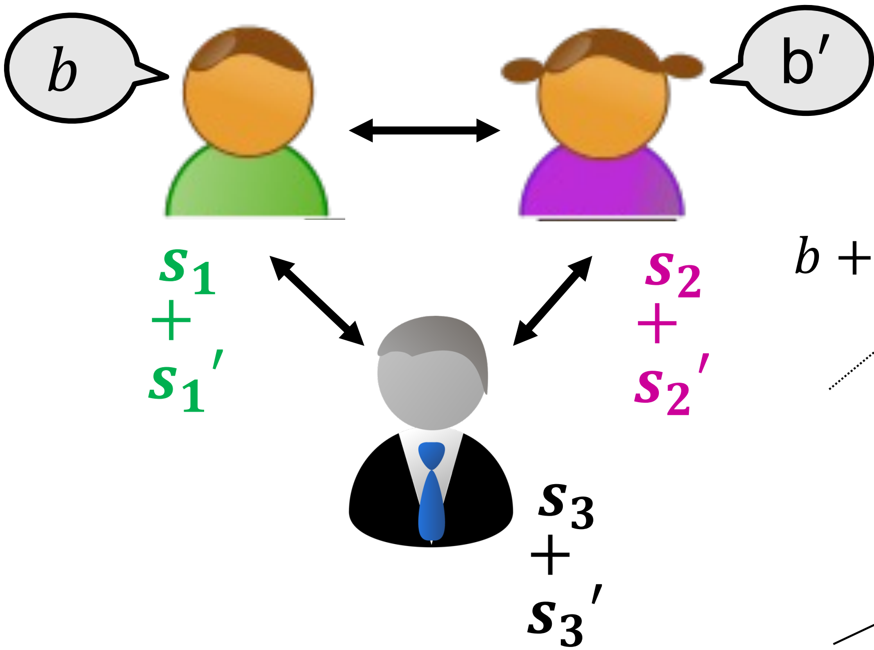
1. Each party secret-shares their input on a degree-t polynomial.

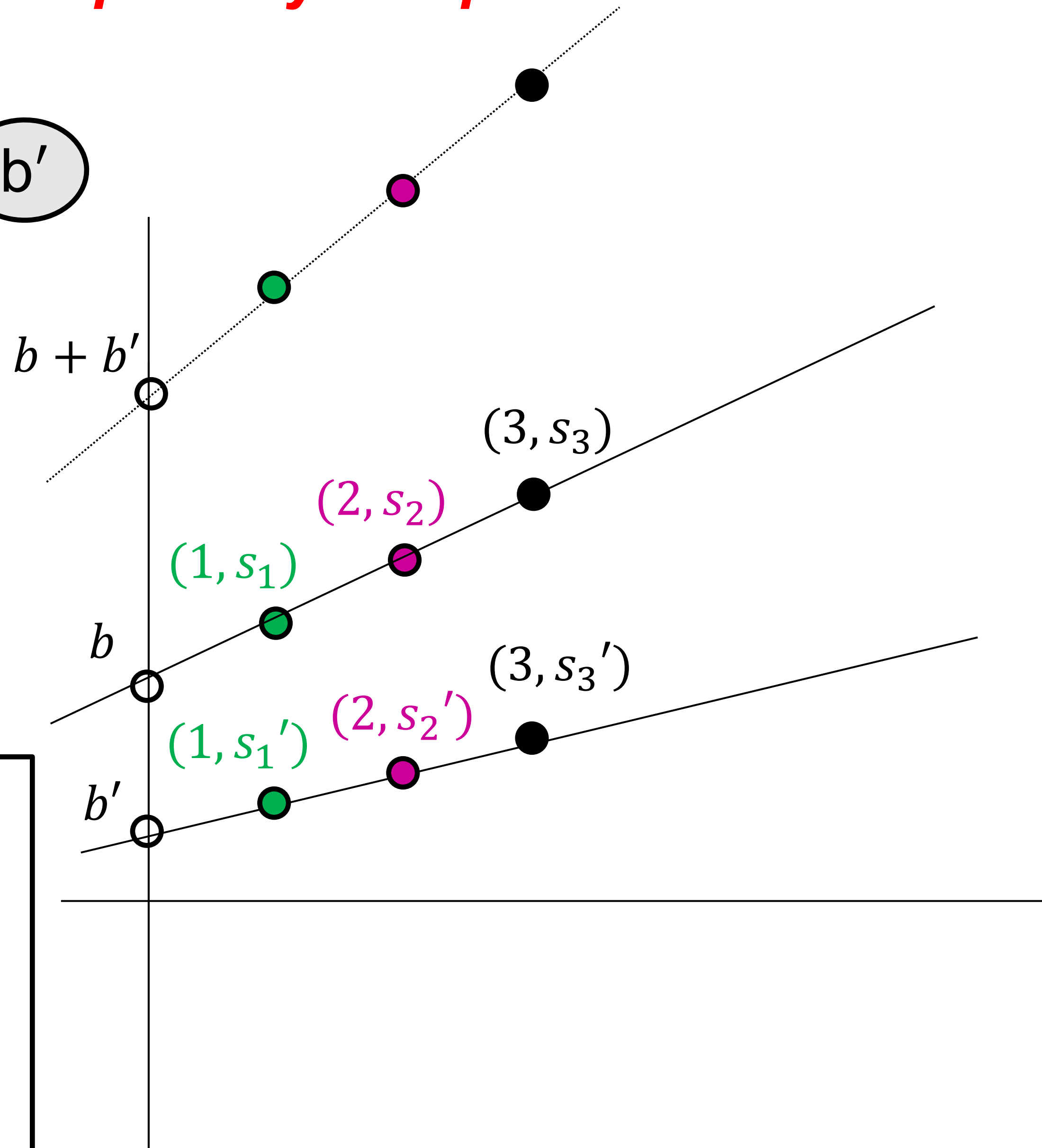
   // so, security against t corruptions

2. Proceed gate by gate, maintaining the invariant that the parties holds a secret sharing of every wire value.
3. Exchange the output shares & reconstruct the output.

Secure Multiparty Computation

Key Insight: Can homomorphically compute on shares!



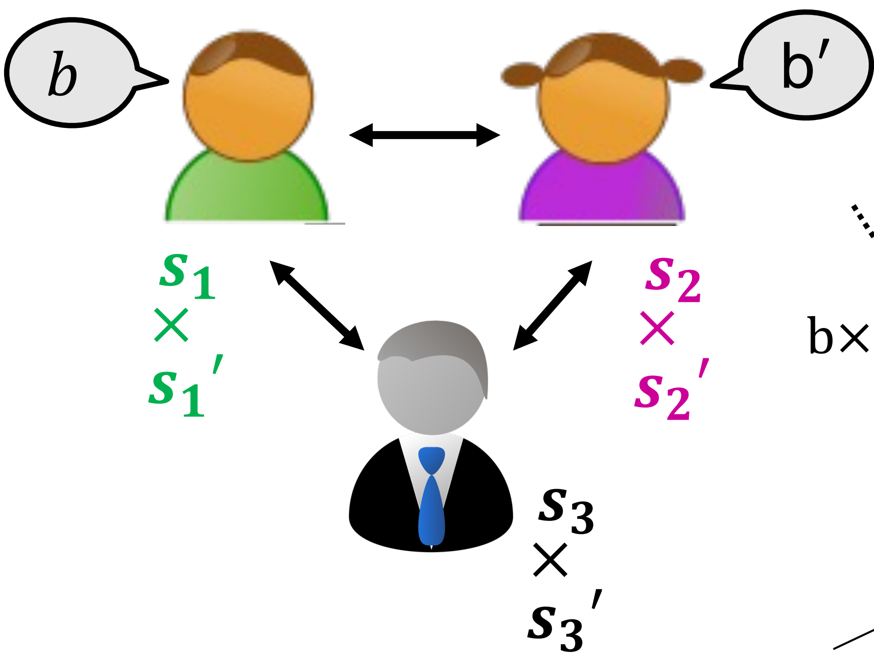


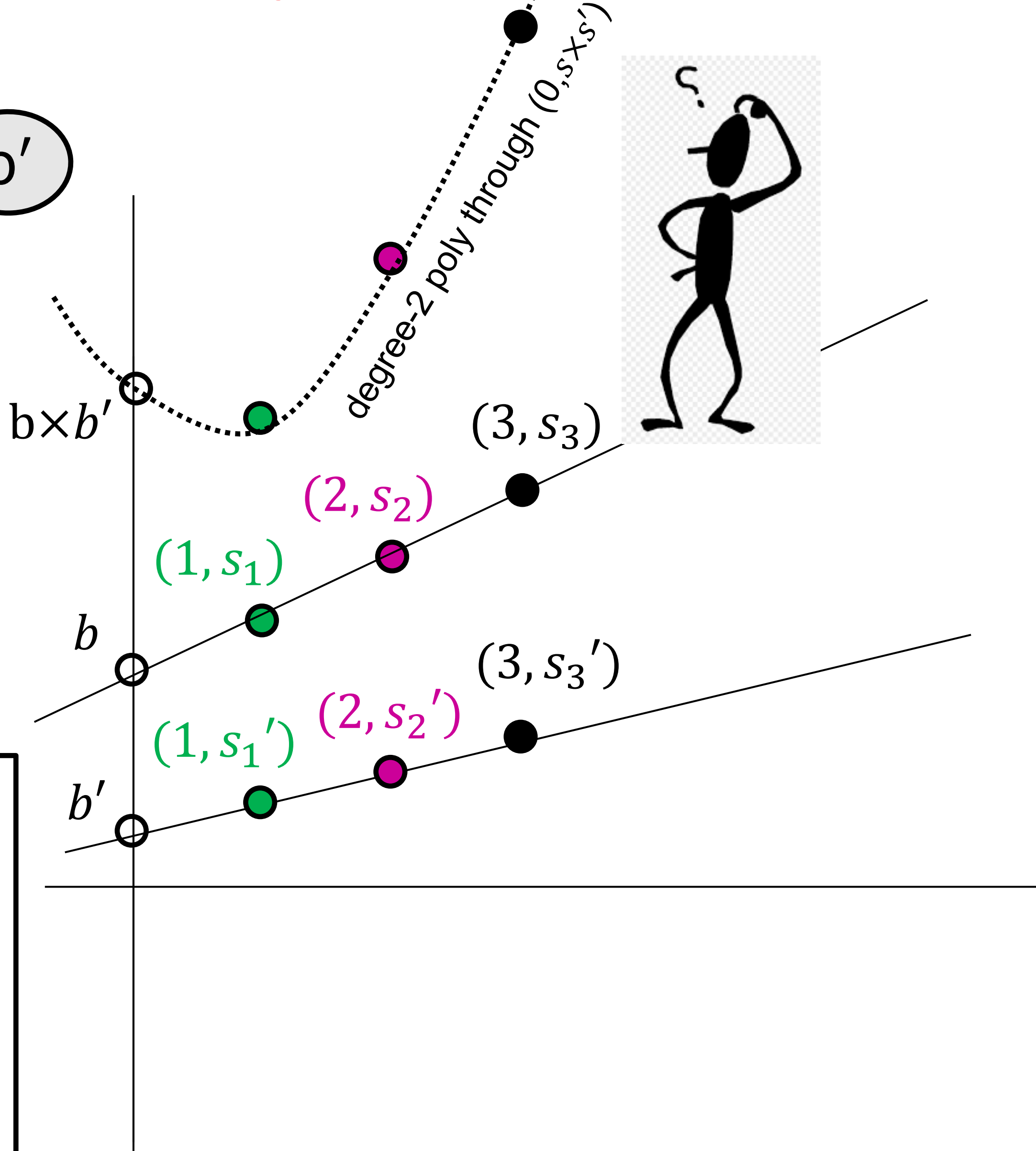
Addition gate:

Locally add shares

Secure Multiparty Computation

Key Insight: Can homomorphically compute on shares!





Multiplication gate:

Locally multiply shares

Multiplication

In general, after a single multiplication, the shares will live on a degree-2t polynomial.

Need $2t + 1$ shares to reconstruct.

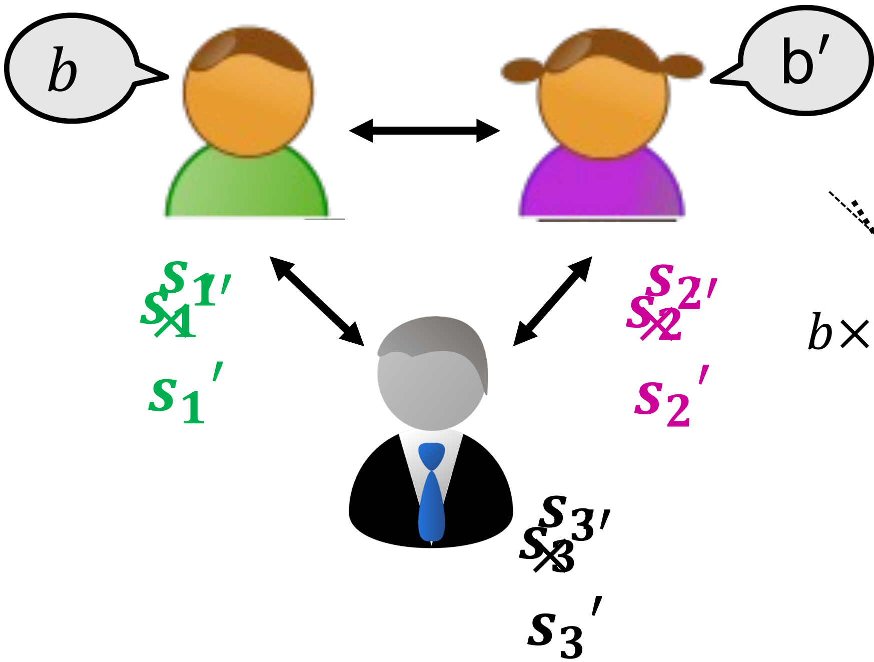
We know that $n > 2t$, so the n shares together have enough information to recover the product of the secrets!

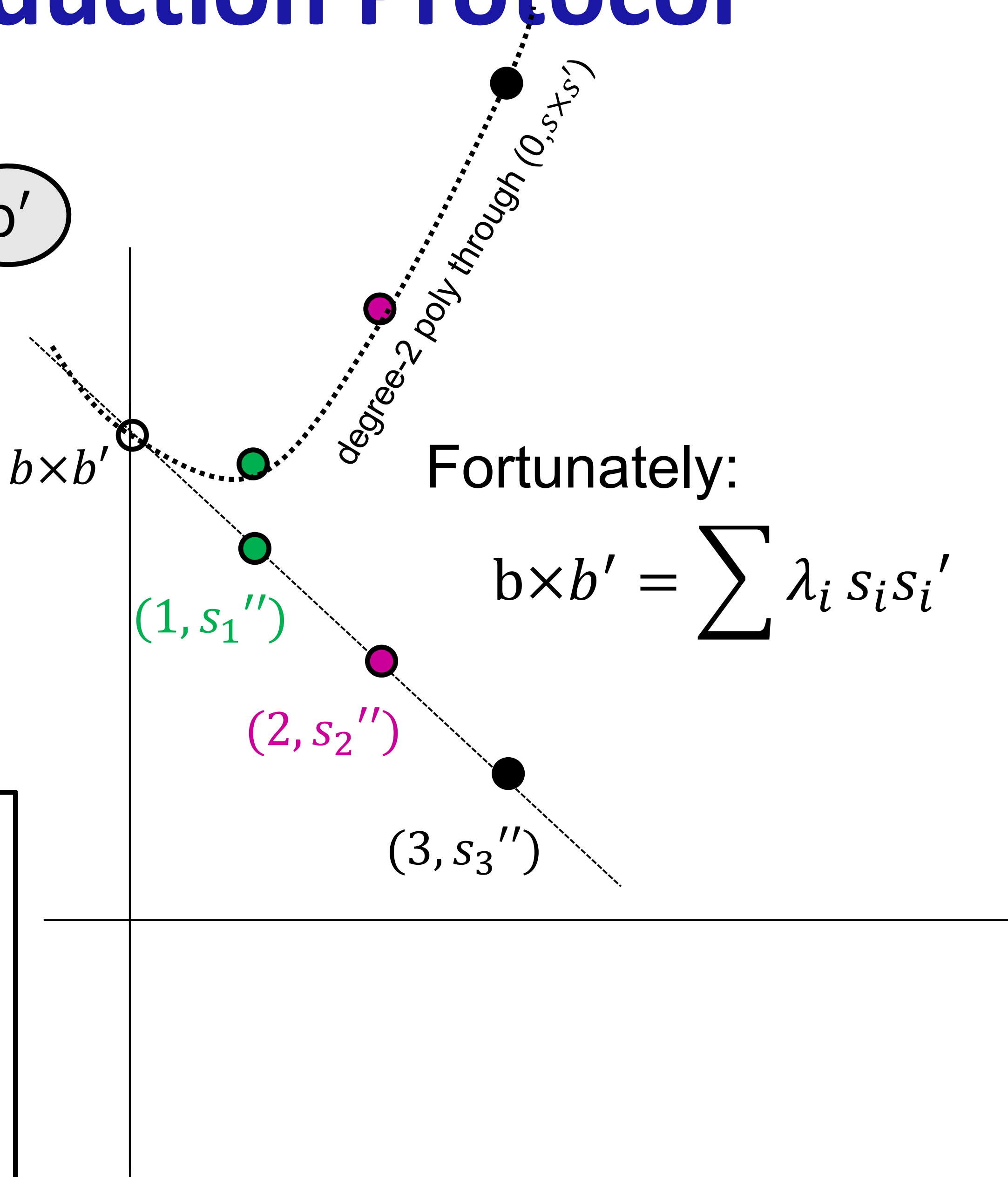
What's more, we also know that this recovery process is a linear function of the shares.

$$\mathrm{b} \times \mathrm{b}' = \sum \lambda_i \, s_i s_i'$$

for some publicly known coefficients λ_i.

Degree Reduction Protocol





Fortunately:

$$\mathrm{b}{\times}b' = \sum \lambda_i \, s_i s_i'$$

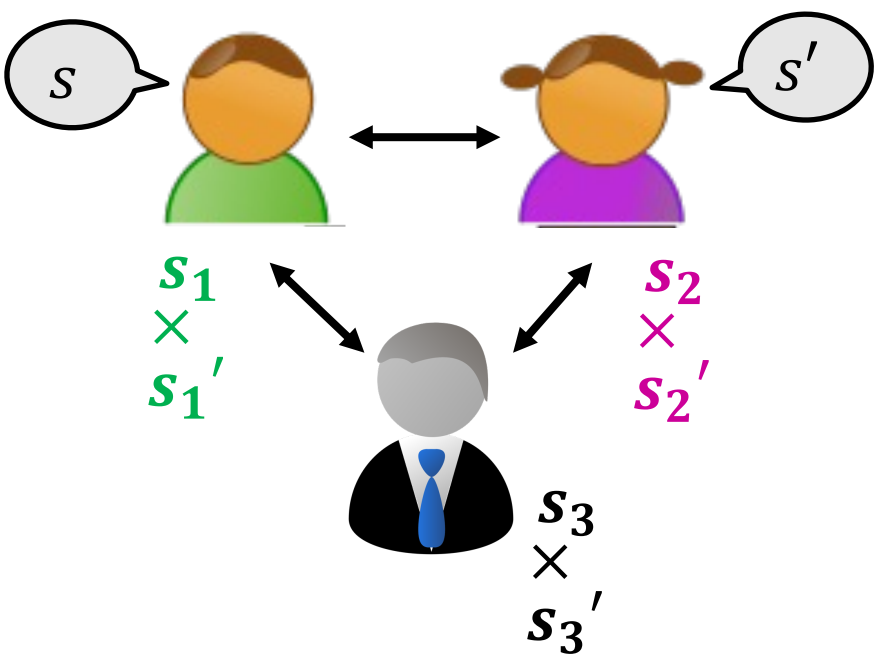
Multiplication gate:

Locally multiply shares & run a degree reduction protocol.

Degree Reduction Protocol

Convert shares on a degree-2t polynomial into shares on a degree-t polynomial

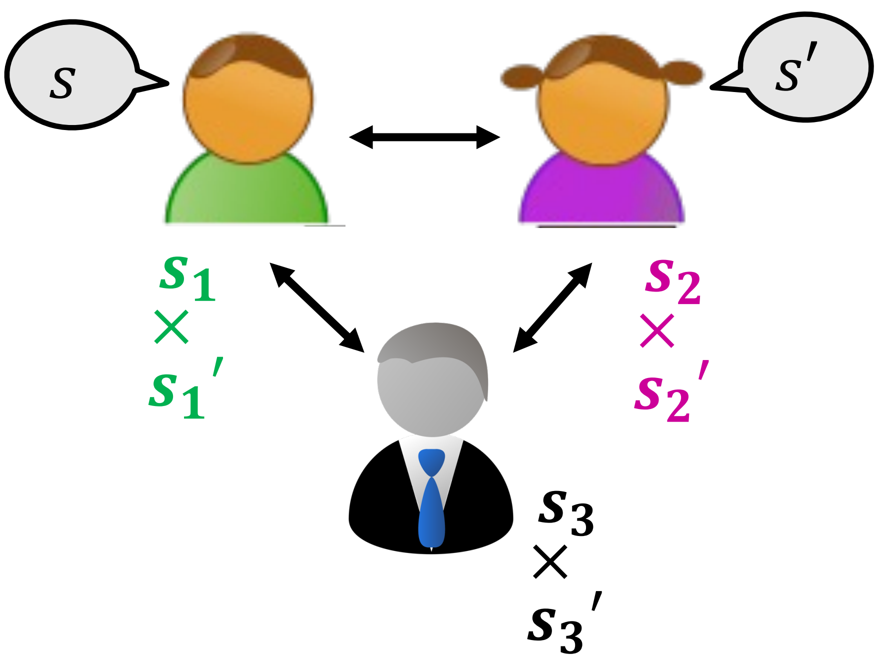
Idea: "homomorphically" compute the **linear function** $\sum \lambda_i * (\cdot)$ on the local product shares.



1. Each party t-out-of-n shares $s_i \times s_i'$ to all parties
2. Each party computes a linear combination of the shares it receives using coefficients λ_i.

This is the moral equivalent of bootstrapping in FHE!

Idea: "homomorphically" compute the **linear function** $\sum \lambda_i * (\cdot)$ on the local product shares.



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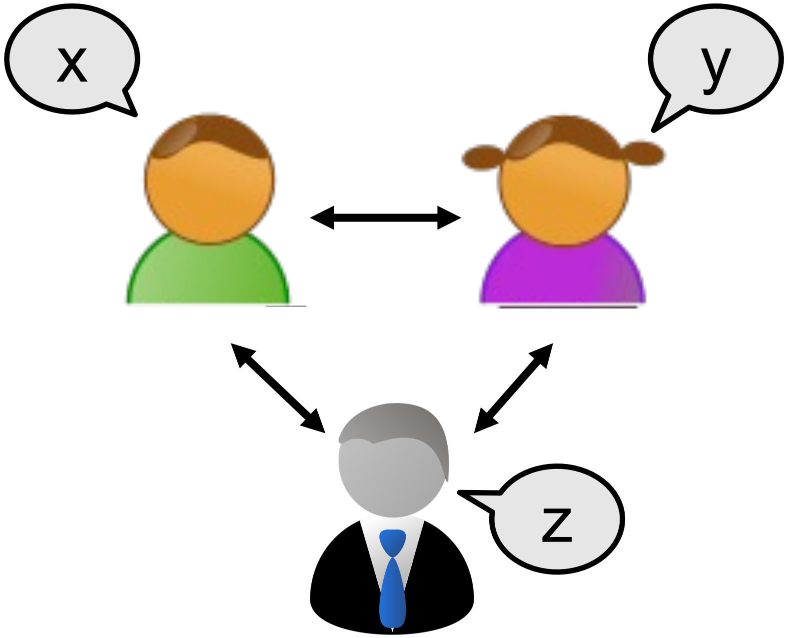
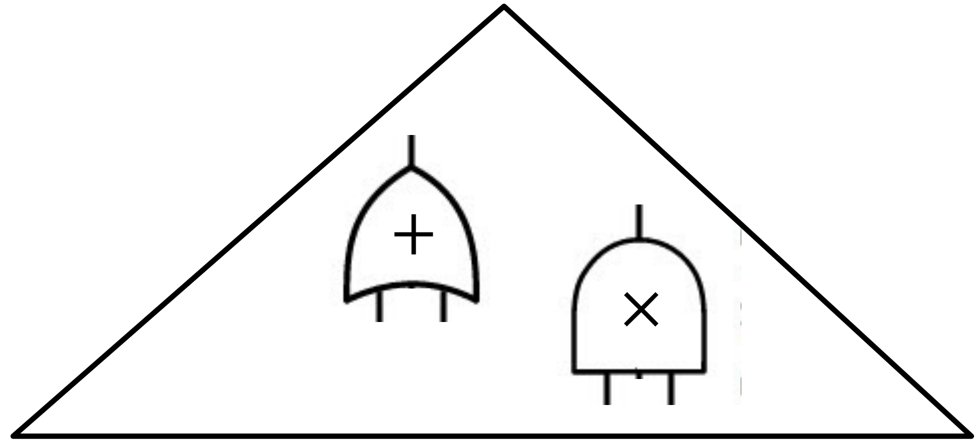
Secure Multiparty Computation





Arithmetic circuit for F

1. Each party secret-shares their input.

2. Proceed gate by gate:
   ADD: locally add shares
   MULT: locally mult shares and do degree reduction.

3. Exchange the output shares & reconstruct the output.

Communication Complexity ∝ #AND gates

Security Intuition

1. Any subset of t parties do not get any information about other parties' inputs from the input shares.

2. Security of the degree-reduction protocol: any subset of t parties sees completely random numbers

3. The output lives on a random polynomial of degree t whose constant term is the circuit output. The shares, therefore, reveal only the circuit output.

Threshold Decryption and Signing

Secret sharing is useful way beyond MPC.

An example: distributed storage of keys.

Another example, threshold decryption:

distributed storage of decryption key + non-interactive distributed (or threshold) decryption

Threshold El Gamal

Public key: g^x

Secret key: x

I am paranoid about losing x so I share it among n servers.

I secret-share x into n shares $x_1, \ldots, x_n$ s.t. $\sum_{i=1}^{n} x_i = x \pmod{q}$

Threshold Decryption:

Given a ciphertext $(g^r, g^{rx}m)$, the servers each compute a decryption share $(g^r)^{x_i}$.

Multiplying the decryption shares gives us $\prod (g^r)^{x_i} = g^{rx}$ which in turn gives us m after division.

Threshold Decryption and Signing

Secret sharing is useful way beyond MPC.

An example: distributed storage of keys.

Another example, threshold decryption:
distributed storage of decryption key + non-interactive distributed (or threshold) decryption

Yet another example, threshold signing.

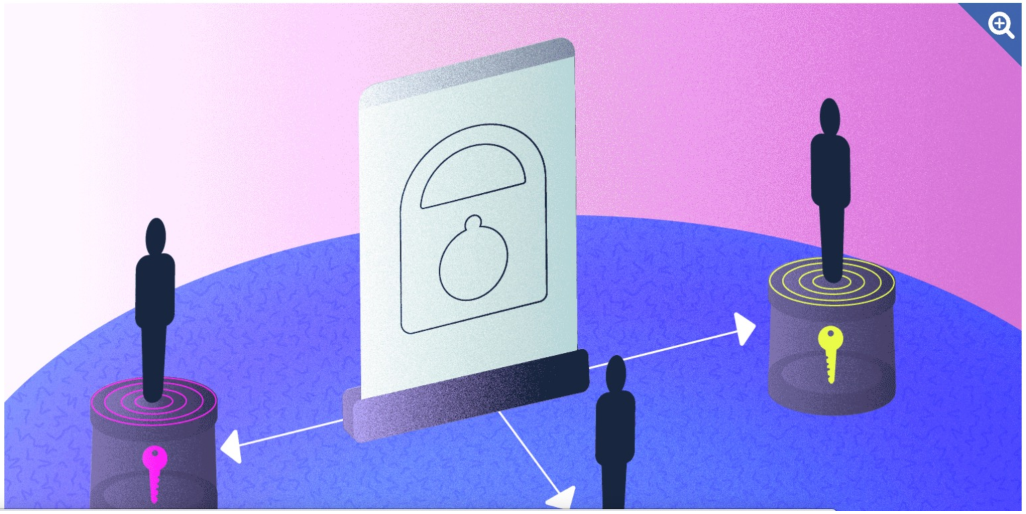
NEWS

NIST Kick-Starts 'Threshold Cryptography' Development Effort

Establishing the emerging technique's building blocks is a near-term focus.

July 07, 2020

Share



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ORGANIZATIONS

- Information Technology Laboratory
  - Computer Security Division
    - Cryptographic Technology Group
    - Security Test, Validation and Measurement Group

PayPal to acquire cryptocurrency security startup Curv

Romain Dillet @romaindillet / 9:44 AM EST • March 8, 2021

Comment

Behind the scenes, Curv uses multi-party computation to handle private keys. When you create a wallet, cryptographic secrets are generated on your device and on Curv's servers. Whenever you're trying to initiate a transaction, multiple secrets are used to generate a full public and private key.

Secrets are rotated regularly and you can't do anything with just one secret. If somebody steals an unsecured laptop, a hacker cannot access crypto funds with the information stored on this device alone.

Coinbase to acquire leading cryptographic security company, Unbound Security

Coinbase Follow

Nov 30 · 3 min read

Today, we're announcing the next phase of our security journey with the acquisition of Unbound Security. Based in Israel, it is a pioneer in a number of cryptographic security technologies, including the emerging field of secure multi-party computation (MPC), a highly advanced technology for which Unbound Security's co-founder, Yehuda Lindell, is a world leader. With this acquisition, Coinbase not only gains access to some of the world's most sophisticated cryptographic security experts, including Unbound Security co-founder and current Vice President of Research and Development, Guy Peer, who brings more than 20 years of experience in cryptographic security, but also a presence in Israel, a well-established and rapidly growing technology hub. This presence in Israel will add an additional powerful prong to Coinbase's global talent acquisition strategy, following on closely to recent thrusts into engineering talent bases such as India, Singapore and Brazil.

Crypto can't grow without strong cryptography and strong security, but it also needs to be user friendly. Secure multi-party computation is an